Review Questions on Power and Taylor Series
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Disclaimer: This is a list of questions to guide you through your studies in Sequences and Series. Not everything that is asked in these questions will appear in the test, and conversely, there might be a question in the test that was not explicitly covered by these questions. Use these questions only as a guide in your studies and don't feel like you need to answer every question to be 100% ready for your test. **Solutions to these questions will not be provided.**

Power Series

1. A **power series** is a series of the form $\sum_{n=0}^{\infty} c_n (x-a)^n$ where the c_n are numbers and a is a fixed constant. We say that this is a power series centered at $x = a$.

2. The **interval of convergence** of a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ centered at $x = a$ is an interval of the form $(a-R, a+R)$ with the property that for every number x in $(a-R, a+R)$ the series $\sum_{n=0}^{\infty} c_n (x-a)^n$ converges.

3. The number R is called the **radius of convergence** and it can be found by using the Ratio Test.

4. What are the three cases for the interval of convergence of a power series?

5. Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{2n}{n!}(x-1)^n$. Let $f(x)$ be the function defined by the power series on its interval of convergence. What is the power series for $f'(x)$? What is the power series for $\int f(x)\, dx$?

6. Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{n}{3^n}(x-4)^n$. Let $f(x)$ be the function defined by the power series on its interval of convergence. What is the power series for $f'(x)$? What is the power series for $\int f(x)\, dx$?

7. Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{n!}{3^n}x^n$. Let $f(x)$ be the function defined by the power series on its interval of convergence. What is the power series for $f'(x)$? What is the power series for $\int f(x)\, dx$?

8. Find a power series representation for the function $f(x) = \dfrac{7}{2-3x}$ and state the interval of convergence of the power series.

9. Find a power series representation for the function $f(x) = 7\arctan(3x)$ and state the interval of convergence of the power series.

10. Find a power series representation for the function $f(x) = 2x\ln(7+x)$ and state the interval of convergence of the power series.

11. Find a power series representation for the function $f(x) = \dfrac{2x}{(1+x)^2}$ and state the interval of convergence of the power series.

Taylor Series

1. Given a function $f(x)$ that has derivatives of all orders at $x = a$, we can generate the power series $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$. This power series is called the **Taylor series of $f(x)$ centered at $x = a$.**

2. What is Taylor's formula? Write it out as accurately as possible.

3. Give an example of a function $f(x)$ whose Taylor series at $x = 0$ does not equal $f(x)$. (We only gave one example and so you should memorize this example.)

4. What is the Maclaurin series of $f(x) = \cos(x)$ and what is its interval of convergence?

5. What is the Maclaurin series of $f(x) = \sin(x)$ and what is its interval of convergence?

6. What is the Maclaurin series of $f(x) = e^x$ and what is its interval of convergence?

7. What is the Taylor series of $f(x) = \ln(x)$ centered at $x = 1$ and what is its interval of convergence?

8. What is the Maclaurin series of $f(x) = \frac{1}{1-x}$ and what is its interval of convergence? What is the Maclaurin series of $f'(x)$ and what is its interval of convergence?

9. What is the Maclaurin series of the function $f(x) = \int \cos(x^2)\, dx$ and what is its interval of convergence?

10. What is the Maclaurin series of the function $f(x) = \int e^{x^2}\, dx$ and what is its interval of convergence?

11. Find a power series representation of $f(x) = x^2 \cos(x/2)$.

12. If $f(x) = \sum_{n=0}^{\infty} \frac{n}{2^n}(x-3)^n$ then what is the domain of $f(x)$? What is the Taylor series of $f(x)$ centered at $x = 3$? What is $f^{(4)}(3)$? (see the last review question)