Review Questions on Sequences and Series

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Disclaimer: This is a list of questions to guide you through your studies in Sequences and Series. Not everything that is asked in these questions will appear in the test, and conversely, there might be a question in the test that was not explicitly covered by these questions. Use these questions only as a guide in your studies and don't feel like you need to answer every question to be 100% ready for your test. **Solutions to these questions will not be provided.**

Use WolframAlpha to Check Your Work
To check your work, you can use WolframAlpha (https://www.wolframalpha.com/) to decide if a sequence converges. For example, to compute $\lim_{n\to\infty} \frac{n+3}{n^2+3}$, enter the following into the WolframAlpha search bar:

```
Limit[ (n+3)/(n^2+3), n -> Infinity]
```

Or to compute $\lim_{n\to\infty} \frac{\ln(n)}{n^2+3}$ type:

```
Limit[ ln(n)/(n^2+3), n -> Infinity]
```

WolframAlpha can also do series. For example, to check the convergence/divergence of $\sum_{n=1}^{\infty} \frac{n+3}{n^3+7}$, enter the following into the WolframAlpha search bar:

```
Sum[ (n+3)/(n^3+7), {n, 1, Inf}]
```

In this case, Wolfram gives an approximation to the actual sum. It also tells you which method it used to decide. On the other hand, Wolfram correctly returns that the series $\sum_{n=1}^{\infty} \frac{n+3}{n^2+7}$ diverges.

Sequences

1. What is a sequence? Use one sentence to answer the question.

2. Is the sequence $a_n = \arctan(n)$ bounded above and/or bounded below? If yes, then find a lower and/or upper bound. You can assume that $n = 0, 1, 2, 3, \ldots$.

3. Is the sequence $a_n = n^2$ bounded above and/or bounded below? If yes, then find a lower and/or upper bound. You can assume that $n = 1, 2, 3, \ldots$.

4. Write the general nth term a_n for the sequence $a = \left(-\frac{7}{2}, \frac{8}{4}, -\frac{9}{8}, \frac{10}{16}, -\frac{11}{32}, \ldots\right)$. Part of your answer must state the initial value of the index n. Does the sequence converge?

5. Write the general nth term b_n for the sequence $b = \left(\frac{1}{3}, \frac{8}{5}, \frac{27}{7}, \frac{64}{9}, \ldots\right)$. Part of your answer must state the initial value of the index n. Does the sequence converge?

6. What is the Squeeze Theorem? Write it out as accurately as possible.

7. Compute $\lim\limits_{n\to\infty} \dfrac{(-1)^n \cos(3n+1)}{n}$, or explain why it does not exist.

8. Compute $\lim\limits_{n\to\infty} \dfrac{n^4(-1)^n}{2^n}$, or explain why it does not exist.

9. Compute $\lim_{n\to\infty} \frac{n^{54}}{2^n}$, or explain why it does not exist.

10. Compute $\lim_{n\to\infty} \frac{n^{54000000}}{2^n}$, or explain why it does not exist.

11. Compute $\lim_{n\to\infty} \frac{1-7n^{11}-56n^{19}+11n^{24}+65n^{33}}{23n^{35}-n^{25}+n^2-6}$, or explain why it does not exist.

12. Compute $\lim_{n\to\infty} \frac{1-7n^{11}-56n^{19}+11n^{24}+65n^{33}}{23n^{33}-n^{25}+n^2-6}$, or explain why it does not exist.

13. Compute $\lim_{n\to\infty} \frac{1-7n^{11}-56n^{19}+11n^{24}+65n^{35}}{23n^{33}-n^{25}+n^2-6}$, or explain why it does not exist.

14. Compute $\lim_{n\to\infty} \frac{1+\sqrt{n}}{n^2+1}$, or explain why it does not exist.

15. Compute $\lim_{n\to\infty} \frac{n^2}{e^{-3n}}$, or explain why it does not exist.

16. Compute $\lim_{n\to\infty} \frac{n3^n}{4^n}$, or explain why it does not exist.

17. Compute $\lim_{n\to\infty} \frac{\sin^2(n^2)}{n^2}$, or explain why it does not exist.

18. Compute $\lim_{n\to\infty} \frac{-2n^3+\sin^2(3n)}{n^3+12}$, or explain why it does not exist.

19. Compute $\lim_{n\to\infty} \left(\frac{18}{11^n}+7\arctan(n^5)\right)$, or explain why it does not exist.

20. Compute $\lim_{n\to\infty} (e^{2n}+6n)^{\frac{1}{n}}$, or explain why it does not exist.

21. Comptue $\lim_{n\to\infty} \frac{\ln(n)}{n^{3/2}}$, or explain why it does not exist.

22. Comptue $\lim_{n\to\infty} \frac{\arctan(n)}{n}$, or explain why it does not exist.

Series

1. What is a series? Use one sentence to answer the question.

2. What is the sequence of partial sums of a series $\sum_{n=1}^{\infty} a_n$?

3. Fill in the blank: The series $\sum a_n$ converges to L if ________________________ converges to L.

4. The sequence of partial sums $\{s_n\}_{n=1}^{\infty}$ of the series $\sum_{n=1}^{\infty} a_n$ is computed to be $s_n = \frac{5n^2+n+1}{2n^2-n+3}$. Does $\sum_{n=1}^{\infty} a_n$ converge? If yes, what does it converge to? Does $\{a_n\}_{n=1}^{\infty}$ converge? If yes, what does it converge to?

5. The sequence of partial sums $\{s_n\}_{n=1}^{\infty}$ of the series $\sum_{n=1}^{\infty} a_n$ is computed to be $s_n = \frac{n+1}{2n^2-n+3}$. Does $\sum_{n=1}^{\infty} a_n$ converge? If yes, what does it converge to? Does $\{a_n\}_{n=1}^{\infty}$ converge? If yes, what does it converge to?

6. The sequence of partial sums $\{s_n\}_{n=1}^{\infty}$ of the series $\sum_{n=1}^{\infty} a_n$ is computed to be $s_n = \frac{n^3+1}{2n^2-n+3}$. Does $\sum_{n=1}^{\infty} a_n$ converge? If yes, what does it converge to? Does $\{a_n\}_{n=1}^{\infty}$ converge? If yes, what does it converge to?

7. Calculate the first 3 terms of the sequence of partial sums of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

8. Calculate the first 3 terms of the sequence of partial sums of the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

9. If $\sum a_n = 11$ and $\sum b_n = -3$ then what is $\sum(2a_n + 7b_n)$?

10. Write out the formula for the Geometric series. Be careful with where the index starts.

11. Find the sum
$$5 + \frac{5}{2} + \frac{5}{2^2} + \frac{5}{2^3} + \frac{5}{2^4} + \cdots$$

12. Find the sum
$$7 + 11 - 33 + 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \cdots$$

13. Find the sum
$$1 - \frac{2}{5} + \frac{4}{25} - \frac{8}{125} + \frac{16}{625} - \cdots$$

14. Find the sum
$$\frac{7^3}{8^3} + \frac{7^4}{8^4} + \frac{7^5}{8^5} + \frac{7^5}{8^5} + \frac{7^6}{8^6} + \cdots$$

15. Find the sum
$$1 + \frac{3}{2} + \frac{3^2}{2^2} + \frac{3^3}{2^3} + \frac{3^4}{2^4} + \frac{3^5}{2^5} + \cdots$$

16. What is a p-series? When does a p-series converge?

17. Write out the Integral Test.

18. Write out the Comparison Test.

19. Write out the Limit Comparison Test.

20. Write out the Absolute Convergence Test.

21. Write out the Ratio Test.

22. Write out the Alternating Series Test.

23. TRUE or FALSE: Let $\sum a_n$ be a given series. If $\lim_{n\to\infty} a_n = 1$, then the series $\sum a_n$ converges to 1.

24. TRUE or FALSE: Let $\sum a_n$ be a given series. If $\lim_{n\to\infty} a_n = 0$, then necessarily the series $\sum a_n$ converges.

25. TRUE or FALSE: If $\sum a_n$ converges then necessarily the sequence of partial sums s_n converges to zero: $\lim_{n\to\infty} s_n = 0$.

26. TRUE or FALSE: If $\sum a_n$ converges then necessarily the sequence a_n converges to zero: $\lim_{n\to\infty} a_n = 0$.

27. TRUE or FALSE: The Integral Test can be applied to the series $\sum a_n$ only when the sequence is non-negative: $a_n \geq 0$.

28. TRUE or FALSE: The Comparison Tests can be applied only to series that are non-negative.

29. TRUE or FALSE: Let $\sum a_n$ be a given series. Suppose that $\sum |a_n|$ converges. Then we can conclude that $\sum 5a_n$ also converges.

30. TRUE or FALSE: Let $\sum a_n$ be a given series. Suppose that $\sum a_n$ converges. Then we can conclude that $\sum |a_n|$ also converges.

31. TRUE or FALSE: The Ratio Test can be applied only to series that are non-negative.

32. TRUE or FALSE: If a series converges absolutely then it must also converge.

33. Construct a series $\sum_{n=0}^{\infty} a_n$ that converges to 5. Hint: Use a Geometric series.

34. Construct a series $\sum_{n=0}^{\infty} a_n$ that converges to $-\pi$. Hint: Use a Geometric series.

35. TRUE or FALSE: Suppose that $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 3$. Then necessarily the series $\sum |a_n|$ converges.

36. TRUE or FALSE: Suppose that $\sum b_n$ converges and suppose that $b_n < a_n$ for all n. Then $\sum a_n$ must also converge.

37. TRUE or FALSE: Suppose that $\sum b_n$ diverges and suppose that $a_n < b_n$ for all n. Then $\sum a_n$ must also diverge.

38. **It is very useful to know that** $\ln(n) < \sqrt{n}$ **for all** $n > 0$. For example, consider $\sum \frac{(\ln(n))^6}{n^5}$. Since $\ln(n) < \sqrt{n}$ then $(\ln n)^6 < (\sqrt{n})^6 = n^3$. Therefore, $\frac{(\ln(n))^6}{n^5} < \frac{n^3}{n^5} = \frac{1}{n^2}$. Because the series $\sum \frac{1}{n^2}$ converges, by the Comparison Test Part (i), then $\sum \frac{(\ln(n))^6}{n^5}$ converges also.

39. Does $\sum_{n=2}^{\infty} \frac{1}{\ln(n^6)}$ converge? Explain/show how you decide.

40. Does $\sum_{n=2}^{\infty} \frac{1}{\ln(2^n)}$ converge? Explain/show how you decide.

41. Does $\sum_{n=2}^{\infty} \frac{1}{\ln(2^{n^3})}$ converge? Explain/show how you decide.

42. Does $\sum_{n=1}^{\infty} \frac{(2n+7)(-1)^n}{3n^3-7n+1}$ converge? Explain/show how you decide.

43. Does $\sum_{n=1}^{\infty} \frac{7-99n^2}{13-3n+100n^2}$ converge? Explain/show how you decide.

44. Does $\sum_{n=1}^{\infty} \frac{7-99n^2}{13-3n+100n^2}$ converge? Explain/show how you decide.

45. Does $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$ converge? Explain/show how you decide.

46. Does $\sum_{n=1}^{\infty} \frac{2n}{e^{n^2}}$ converge? Explain/show how you decide.

47. Does $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ converge? Explain/show how you decide.

48. Does $\sum_{n=1}^{\infty} \frac{4^n}{n!}$ converge? Explain/show how you decide.

49. Does $\sum_{n=1}^{\infty} \frac{10^n(-1)^n}{n4^{2n+1}}$ converge? Explain/show how you decide.

50. Does $\sum_{n=1}^{\infty} \frac{(2n)!}{2^n n!}$ converge? Explain/show how you decide.

51. Does $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n \sqrt{n}}{n!}$ converge? Explain/show how you decide.

52. Does $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$ converge? Explain/show how you decide.