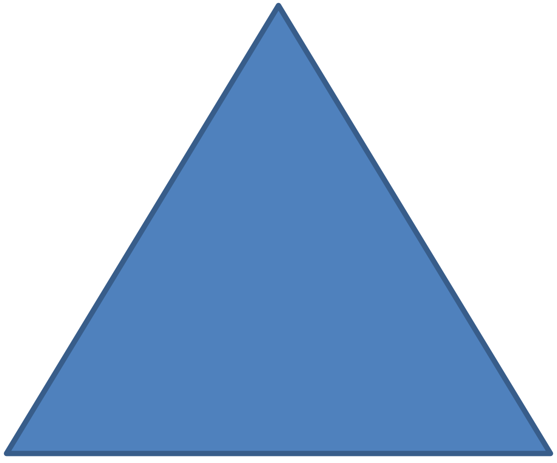
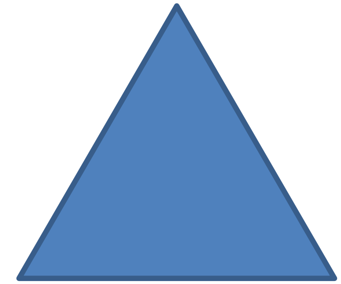
Introduction to Groups

Symmetries of an Equilateral Triangle



Rotational Symmetry

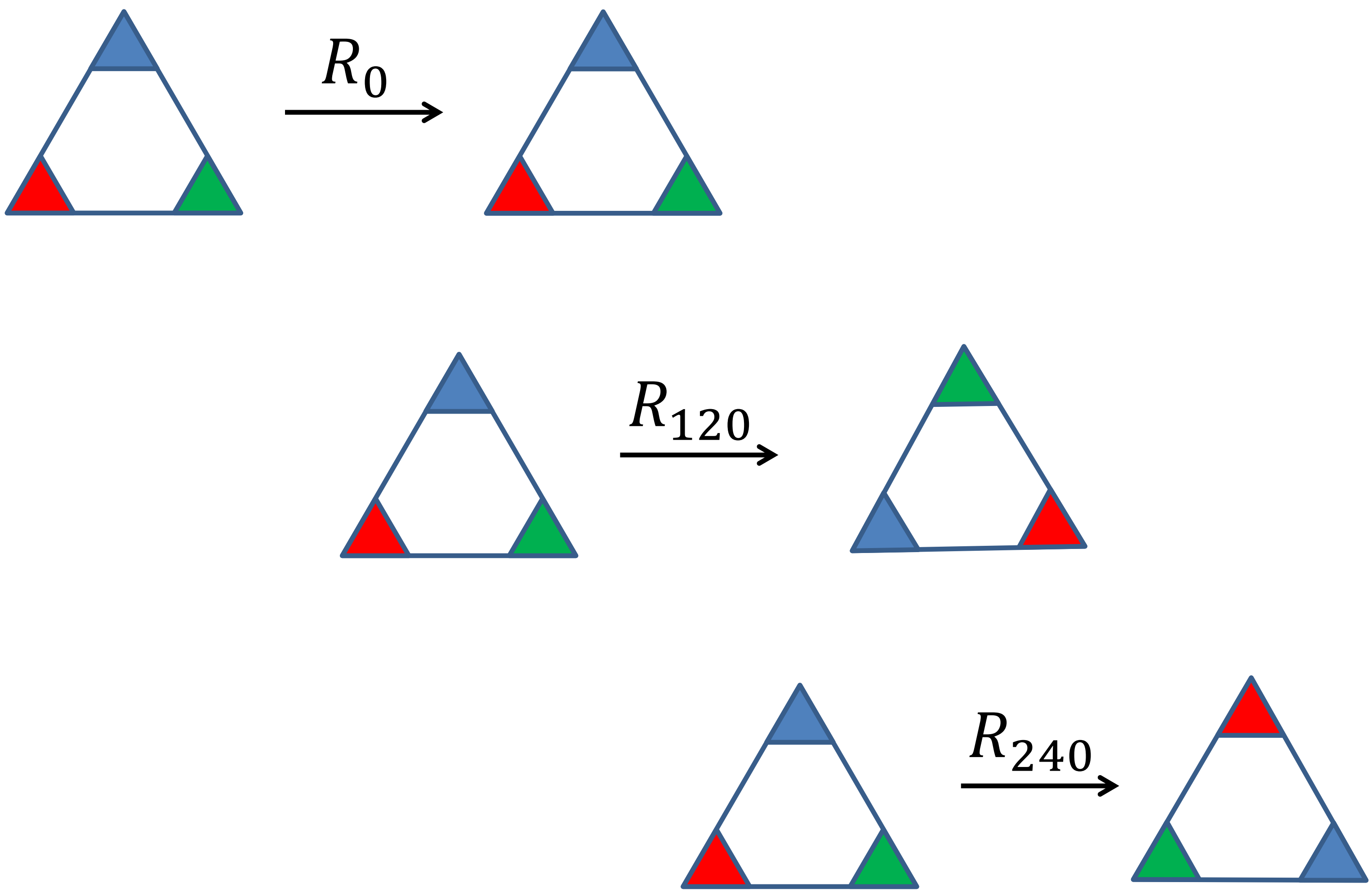


- Let R_θ be a *counterclockwise* rotation of θ degrees.

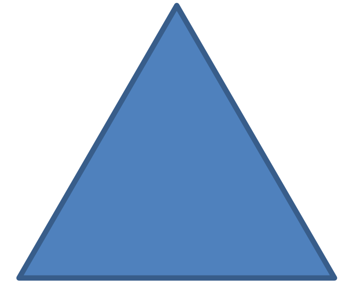
- What are the possible symmetry rotations of an equilateral triangle?

$$R_0\ , R_{120}, \text{ and } R_{240}$$

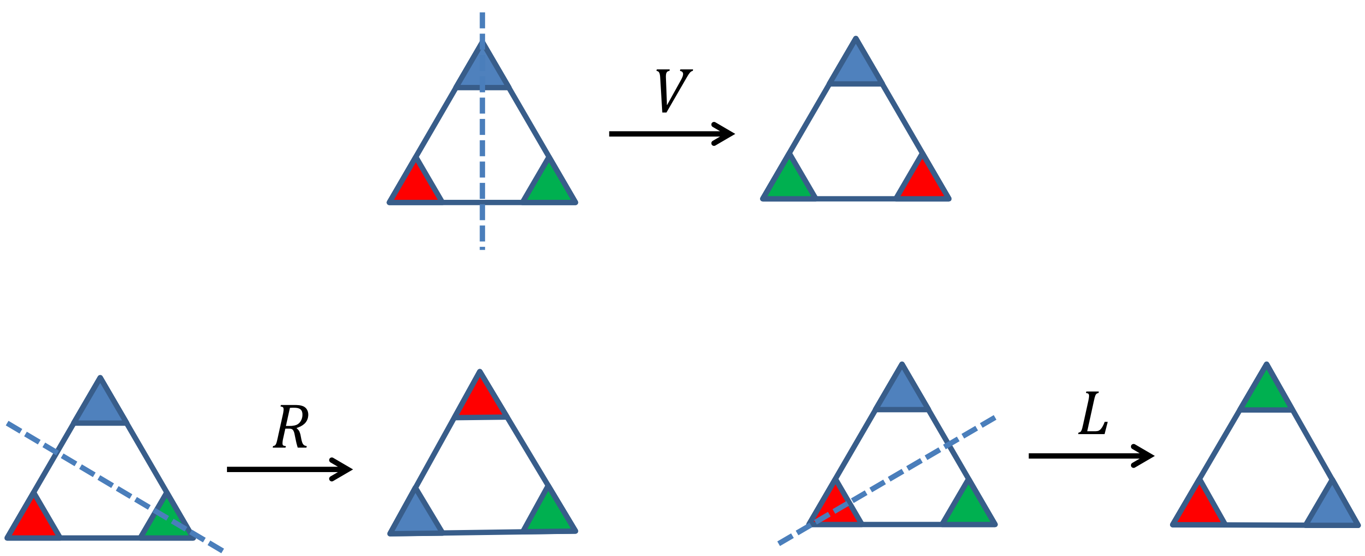
Rotations



Reflective Symmetry

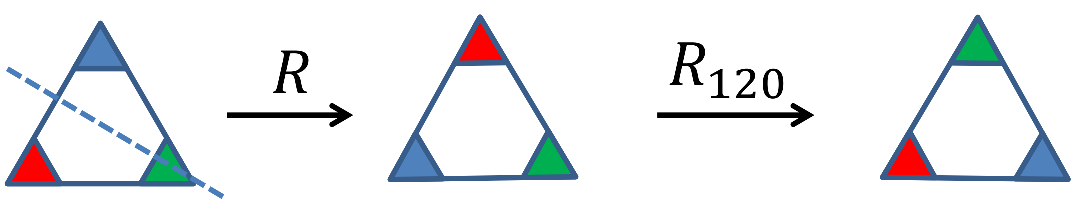


What are the possible reflections of an equilateral triangle?



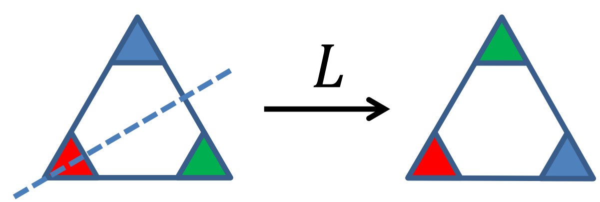
Composition

We can compose (or combine or multiply) the 6 symmetries, but this will always be equivalent to one of the original 6.



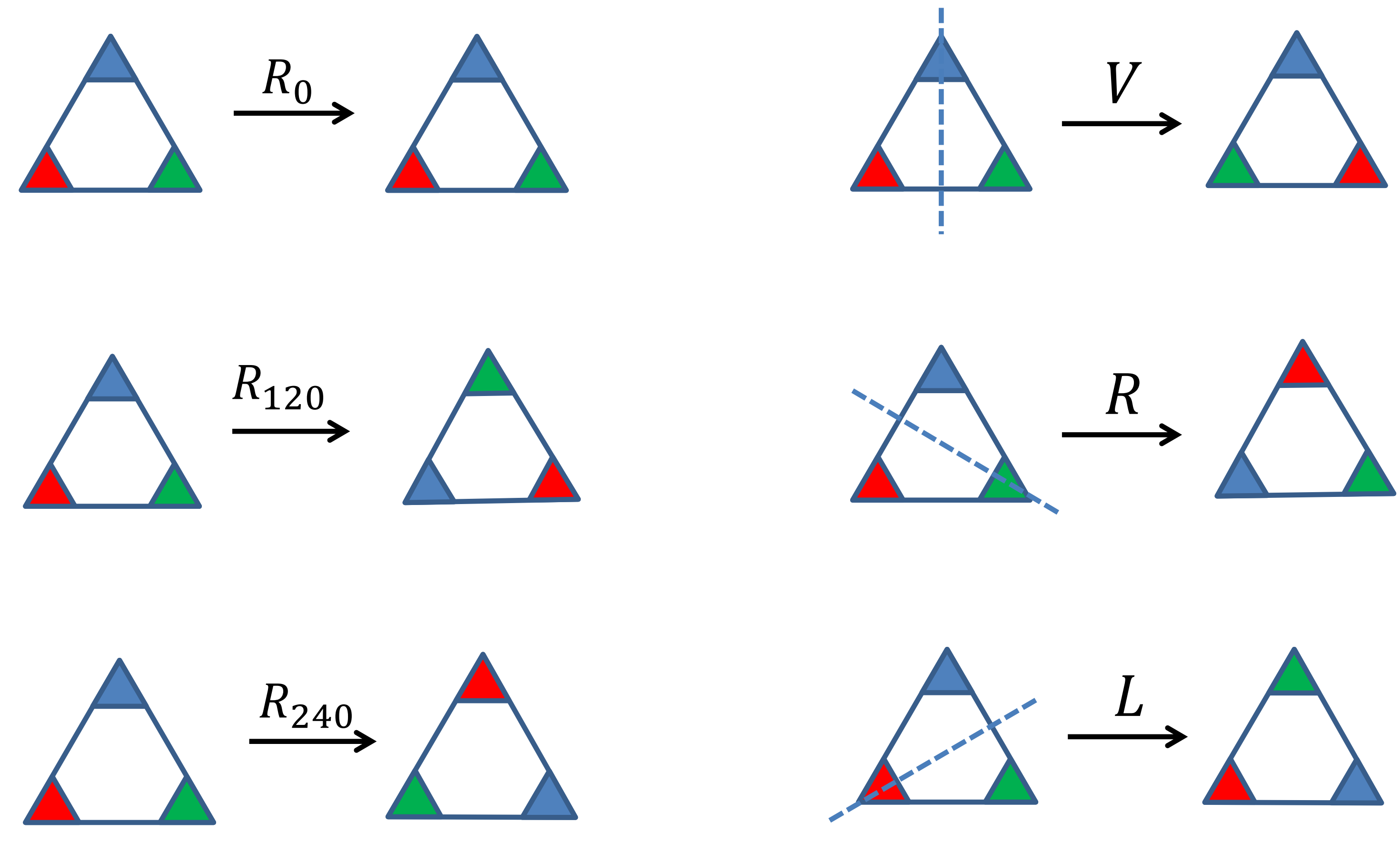
Which of the original 6 is this equivalent to?

This is the same as L.



So we can say $R_{120} \cdot R = L$

Summary of Symmetry



Cayley Table

We can summarize these computations in a Cayley table (or operation table.)

b

$a \cdot b$	R_0	R_{120}	R_{240}	V	R	L
R_0	R_0	R_{120}	R_{240}	V	R	L
R_{120}	R_{120}	R_{240}	R_0	R	L	V
R_{240}	R_{240}	R_0	R_{120}	L	V	R
V	V	L	R	R_0	R_{240}	R_{120}
R	R	V	L	R_{120}	R_0	R_{240}
L	L	R	V	R_{240}	R_{120}	R_0

a

$a \cdot b$	R_0	R_{120}	R_{240}	V	R	L
R_0	R_0	R_{120}	R_{240}	V	R	L
R_{120}	R_{120}	R_{240}	R_0	R	L	V
R_{240}	R_{240}	R_0	R_{120}	L	V	R
V	V	L	R	R_0	R_{240}	R_{120}
R	R	V	L	R_{120}	R_0	R_{240}
L	L	R	V	R_{240}	R_{120}	R_0

This is an example of a group. It is called the "*Dihedral Group of order 6*" and is denoted by D_3.

Notice:

1. Each composition is one of the original 6 symmetries.

(***Closure***: If *A* and *B* are in D_3, then so is *AB*.)

2. R_0 acts as an ***identity***: $R_0 \cdot A = A \cdot R_0 = A$

3. Every element has an ***inverse***: $V \cdot V = R_0$ and

$$R_{120} \cdot R_{240} = R_{240} \cdot R_{120} = R_0$$

4. The compositions in D_3 are ***associative***: $(AB)C = A(BC)$

5. Is the operation ***commutative***?

No, this is an example of a ***nonabelian*** group. If a group has commutativity, it is an ***abelian*** group.