Math 333 - Linear Algebra

Exam 2 Review

Fall 2007

1 Sections Covered

The exam will cover Sections 2.1-2.5 from the textbook. You should read and understand all of these sections.

2 Definitions

You should know ALL definitions covered in the above mentioned sections. Pay special attention to the definitions of each of the following terms. DEFINITELY KNOW (i.e. MEMORIZE) THESE DEFINITIONS.... Also, if asked to give a definition of something, you can NOT give a theorem. For example, the definition of a transformation T being one-to-one is NOT null$(T) = \{0\}$. This is a theorem, not the definition.

Chapter 2: Linear transformation, Null space, Image, Nullity, Rank, One-to-one, Onto, Ordered basis, Matrix representation of linear transformation, Coordinate vector, Inverse of a transformation, Inverse of matrix, Isomorphism, Change of basis matrix.

3 Theorems

You should know the statements and understand the usefulness of all theorems in the above mentioned sections. Pay special attention to each of the following theorems. You will NOT be asked to write the precise statements of most of these theorems, so don't attempt to memorize exactly what they say. Simply know how to use them. However, you may be asked to write the precise statement if it has a ** next to it.

Chapter 2: Theorems 2.1**, 2.2**, 2.3**, 2.4**, 2.11, 2.12, 2.14, 2.15, 2.19, 2.20, 2.22, 2.23

4 Computations and other Exercises

You are responsible for knowing any homework exercise that has been assigned as well as similar computational exercises. However, you may choose to focus on the following types.

1. Determine bases of the nullspace and image of linear transformations $T : V \to W$. Also find nullity(T) and rank(T), and determine if T is linear, one-to-one, or onto.

2. Compute the matrix representation of a linear transformation and the coordinate vector of a vector with respect to given ordered bases.

3. Determine if a linear transformations T is invertible or an isomorphism. You don't have to FIND the inverse of a transformation.

4. Be able to find the change of basis matrix.

5. In addition to the above types of computational exercises, also be able to do the following exercises from the textbook: Sec. 2.1: (17), Sec. 2.3: (7, 11, 12), Sec. 2.4: (4, 5, 6, 7)

5 Proofs

You should be familiar with the various proof techniques we learned in class so that you will be able to do proofs and justifications similar to the homework problems. For example, how do you show that one set is contained in another set, how do you show that two sets are equal, or how do you show something is unique? We went over several different methods, so review them carefully.

You should also be familiar with the proofs of each of the following theorems. Let V and W be vector spaces and $T : V \to W$ be a linear transformation.

1. **Theorem 2.1:** The sets null(T) and image(T) are subspaces of V and W, respectively.

2. **Theorem 2.2:** Let β be a basis of V. Then the set $T(\beta)$ is a generating set for image(T).

3. **Theorem 2.4:** T is one-to-one if and only if null(T) = $\{0\}$.

6 Good Luck!

Good luck on the exam! A lot of the work we have done has been computational, so make sure you can do all of the computational exercises from the homework. Make sure you practice enough of them so that the calculations can be done quickly (and *correctly*). The theoretical portion of the exam will probably be the more difficult aspect simply because the ideas are somewhat new to you. A lot of it is on this review sheet as well as the practice exam. Remember that the point of this course is really to help you become more comfortable with proofs because you should already know how to do most of the computations. With enough practice, you should be fine.

You have all done well so far this semester. Keep up the good work, and don't get stressed! Study thoroughly and you should be okay. Good luck!