213 Problem Set 10 Solutions

4.6 17. We are asked for the intervals of increase and decrease of the original function. We are given the graph for the *derivative*. The most important part to remember is that when the *derivative* is *positive* the *function* is *increasing* and when the *derivative* is *negaitive* the *function* is *decreasing*. Look at the given graph for f', the *derivative*. Let's do negative first, because there's an important point. The graph is only negative (below the x-axis) on the left side, where $x < -1$. So, the only time the function is *decreasing* is for $x < -1$, i.e $(-\infty, -1)$. The short answer is that it is increasing everywhere else. You may say "but it's zero sometimes", yes, but only at that one point which is not enough to stop it from increasing. If you care, this also means there is an absolute minimum at $x = -1$, although this wasn't asked. Some will put the intervals of increase as $(-1, 0)$ and $(0, 1)$ and $(1, \infty)$. I'm not going to deduct for that, but it doesn't stop increasing by being zero only at a point. The better answer for increase is $(-1, \infty)$.

4.6 35. We're looking for information about increasing and decreasing as well as concavity, so we need to know the first and second derivatives. The original function is $f(x) = x + x^2 - x^3$, the first derivative is $f'(x) = 1 + 2x - 3x^2$ and the second derivative is $f''(x) = 2 - 6x$. Our next step is to find zeroes of the derivatives. Factoring is a bit tricky for the derivative, but factoring out -1 helps to give us $-1(3x^2-2x-1) = -1(3x+1)(x-1)$. We then look for when $3x + 1 = 0$ and $x - 1 = 0$. The first happens when $x = -\frac{1}{3}$ and the second when $x = 1$. We can check points between, e.g. -1, 0, and 2 to see if it is increasing or decreasing. We check using the derivative. $f'(-1) = -4$, so the interval $(-\infty, -\frac{1}{3})$ is decreasing for f. $f'(0) = 1$, so the interval $(-\frac{1}{3}, 1)$ is increasing for f. And $f'(2) = -7$, so the interval $(1, \infty)$ is decreasing for f. f goes from decreasing to increasing at $x = -\frac{1}{3}$, so there is a local minimum there where the function equals $f(-\frac{1}{3}) = -\frac{5}{27}$. f goes from increasing to decreasing at 1, so there is a local maximum there where the function equals $f(1) = 1$.

What about concavity? We return to the second derivative $f''(x) = 2 - 6x$, this equals zero at $x = \frac{1}{3}$. This time we can easily check just 0 and 1. $f''(0) = 2$ so f is concave up on $(-\infty, \frac{1}{3})$. And $f''(1) = -4$ so f is concave down on $(\frac{1}{3}, \infty)$. There is an infection point for f at $x = \frac{1}{3}$ where the function is $f(\frac{1}{3}) = \frac{11}{27}$.