213 Problem Set 11 Solutions

4.8.10. I think we did this for cows. It should be much the same. Draw a rectangle, label its dimensions. I like b and h. And one of the b will be covered by the river. We seek to maximise the area, The area is $A = bh$. The fencing is $800ft = b + 2h$ (only one b because of the river). We can most easily solve this for $b = 800 - 2h$, and then substitute back into $A = h(800 - 2h) = 800h - 2h^2$. We have, as always $b \geq 0$ and $h \geq 0$. When $b = 0$, $h = 400$. So our domain for h is $0 \leq h \leq 400$. We also need to know where the derivative is zero $A' = 800 - 4h = 0$. This happens when $h = 200$. So, we find the area for these three values, 0, 200, and 400. Yes, by now we guess the answer is at 200 ft. $A(0) = 0$, $A(200) = 200(800 - 400) = 80,000$, and $A(400) = 400(800 - 800) = 0$. So, as expected the maximum is at $h = 200$ ft, $b = 800 - 2(200) = 400$ ft. The question asked for dimensions, and so we answer the dimensions are 400 ft for the base and 200 ft for the height to give the maximum area.

4.8.22 Good news - it's about money! Less good news, lots of variables. You probably know this better than I do, but I'm guessing that Profit = Revenue - Cost. I think that is the total right now, since $R(n) = an$ and that makes sense if you sell them at a\$ each. I am using n for *number* of pizzas. So, $P(n) = R(n) - C(n)$ is the total profit function. We are asked for the largest profit *per pizza*. The profit per pizza is $Q(n) = \frac{P(n)}{n}$ (Q for quotient). We are given $R(n) = an$ and $C(n) = b + cn + dn^2$. So, $Q(n) = \frac{an-b-cn-dn^2}{n} = a - c - dn - \frac{b}{n}$. There is only one variable, as that part will be easy, and we don't need to worry about it. The domain is also pretty simple ... you can make any number of pizzas, including none. But. if you make none, the proift per pizza is confusing at best, at $\frac{0}{0}$. I'm guessing that isn't what is meant. So, we will consider only positive values for n, although they could be less than one, I guess you could only make one slice. So, our only remaining work is to find the derivative $Q'(n) = -d + \frac{b}{n^2}$. This equals zero when $n = \sqrt{\frac{b}{d}}$, which among other things says that b and d must positive. I think b is some kind of base cost, so it can't be negative. The other variable d is some kind of increasing costs, which feels odd, something like the more you make the more you pay to make each. That feels as if it shouldn't be positive, but ok. If b increases then this increases, which makes sense, if the base cost goes up, then it takes a higher volume needed to offset them. If d is larger then this goes down, which makes sense, if you need to pay extra more for more to make, then your profit will go down the more that you make. I hope you do all read these solutions. This is one of the ways this is actually practical, and I hope you can see that some of this reasoning fits with what you know about money.

4.8.43 We are nice to cats. I was really curious about this and uncertain, but I think the question is *almost* clear. It tells us the cost for the sides but not the top. I think this is because the top is open. I thought I would want to make this with the front open, but apparently they want the top open. I *do* believe that this affects the answer. If you decide to close the top, do you use the bottom or side material to do it? I am almost certain you use the side material, but then again, you could make a plush top to the box. I think, I'm going to say this ... if you don't open the top, that's ok, but you need to have explained how you made that choice. Most importantly, you can't close *all* sides, that is not a box for a cat, that is just a box without a cat.

We are told that it has a square base. I will call the dimensions b for the side of the base, and h for the height of the box. The volume is then $V = b^2h$. The cost is the cost of the base \$$5b^2$ plus the cost of the sides 4(\$$2bh$) (there are four sides). So $C = 5b^2 + 8bh$. The total volume is 4 ft^3. So, $b^2h = 4$. We have two variables, so we solve for one of them. Solving for h is easier and it only occurs once, so that's the best choice. $h = \frac{4}{b^2}$. We put that back into C to get $C = 5b^2 + 8b\left(\frac{4}{b^2}\right) = 5b^2 + \frac{32}{b}$. This is the answer in the book, so, that's comforting. But, we're not done. We haven't found the dimensions yet. What about domain? Much like our examples from Tuesday, having either variable zero gives no volume, so they aren't feasible, so as long as b is positive, we have something to include. We also don't need to check endpoints, so that's helpful. So, now we need the derivative. $C' = 10b - \frac{32}{b^2}$. Setting this equal to zero gives $10b = \frac{32}{b^2}$ and $b^3 = 3.2$. Taking a cube root gives $b \simeq 1.4736$ feet. This also produces $h = 1.842$ feet in height. This feels ... rather small and steep. It's as if the cat is sleeping at the bottom of a tube. Maybe ... being the cheapest is not the best for our cat. I propose spending some more to make it larger in base, and shorter.

How about 2 feet in base and 1 foot high. I'm now curious, what is the cost difference? The first one costs $C = 5(1.4736)^2 + \frac{32}{1.4736} = \$32.57$. The second one, the one I like, costs $C = 10(2) - \frac{32}{2} = \$36$. I'm willing to spend the extra money for the cat, and anyway it's easier to cut the sides at two feet and one foot. So there.