213 Problem Set 12 Solutions

This one should be shorter than recent.

5.4.2. We really only need to notice one idea here: $\frac{d}{dx}e^{ax} = ae^{ax}$. That's it. So, when we take derivatives of e^{ax} we *multiply* by an extra a. Integrating is backwards, so we divide, see $\frac{d}{dx}\left(\frac{1}{a}e^{ax}\right) = e^{ax}$. And so in general $\int e^{ax}dx = \left(\frac{1}{a}e^{ax}\right) + C$ and that should be all we need. So, to the problem at hand: $\int(e^{2x} - \frac{1}{2}e^{x/2})dx = \frac{1}{2}e^{2x} - \frac{1}{2}2e^{x/2} + C = \frac{1}{2}e^{2x} - e^{x/2} + C$.

5.4.4. I think this one is even easier, it only takes one tiny thought, then it's standard. $\int \frac{x-1}{x^2}dx$. Division is unpleasant in derivatives and worse in integrals, but ... we can divide first, and then we'll be all set. Notice $\frac{x-1}{x^2} = \frac{x}{x^2} - \frac{1}{x^2} = \frac{1}{x} - \frac{1}{x^2} = \frac{1}{x} - x^{-2}$. And that's a form we can integrate, so let's wrap this up: $\int \frac{x-1}{x^2}dx = \int \frac{1}{x} - x^{-2}dx. = \ln x + x^{-1} + C = \ln x + \frac{1}{x} + C$.

Yep, that's pretty short.