213 Problem Set 13 Solutions

I'm kinda convinced that no one reads these. So, the first person to email me saying "I saw this in the solutions" will earn 2 extra points on this problem set. It's lucky number thirteen!

5.5.5. We're given the answer, and the steps. So, this is likely quite confident. We are told to check that $\int x\sqrt{4x^2+9}dx = \frac{1}{12}(4x^2+9)^{3/2}+C$. To do this, we take the derivative of (differentiate) the right side, $\frac{1}{12}(4x^2+9)^{3/2}+C$. The derivative is $\frac{1}{12}\frac{3}{2}(4x^2+9)^{1/2}8x$ which cleans up to the desired $x\sqrt{4x^2+9}$. We're not surprised.

Now we try it out with substitution. As suggested $u = 4x^2+9$, $du = 8xdx$, and hence $dx = \frac{1}{8x}du$. Putting these in gives: $\int x\sqrt{4x^2+9}dx = \int x\sqrt{u}\frac{1}{8x}du = \int \frac{1}{8}u^{1/2}du$. The middle step is not quite formally correct, but it helps you to see how we got there. Now we integrate: $= \frac{2}{3}\frac{1}{8}u^{3/2}+C = \frac{1}{12}(4x^2+9)^{3/2}+C$ as expected.

5.5.19. I'm not sure why they say "change of variables" here instead of "substitution", in any case I think both are the same so I will do as we find familiar. $\int t(1-t^2)^{10}dt$. Let $u = 1-t^2$, the inside. By the way we *could* distribute this, but that would be a lot of work. A machine would be happy doing it that way. We're ... not. Back to the story, we differentiate to find $du = -2tdt$, or $dt = -\frac{1}{2t}du$. Substituting, again in two steps, we get: $\int t(1-t^2)^{10}dt = \int -tu^{10}\frac{1}{2t}du = \int -\frac{1}{2}u^{10}du$ which we hope is easy to integrate to get $-\frac{1}{22}u^{11}+C = -\frac{1}{22}(1-t^2)^{11}+C$. Not too bad, I hope.