213 Problem Set 3 Solutions

§2.4.14. We're considering $\lim_{h\to 0} \frac{(1+h)^2-1}{h}$. This example will start looking very familiar in the coming weeks. The first directions say to simply substitute $h = 0$, so we get $\frac{(1+0)^2-1}{0} = \frac{0}{0}$. That's as expected, but needed as it was asked for. So, let's do some little algebra:

$$\frac{(1+h)^2-1}{h} = \frac{(1+h)(1+h)-1}{h} = \frac{1+2h+h^2-1}{h} = \frac{2h+h^2}{h} = \frac{h(2+h)}{h} = 2+h$$

The last equality is true unless $h = 0$, which is good because we want the limit approaching zero, not at zero. When we now check that, we get $2 + 0 = 2$.

§2.4.32. This one is even simpler. When the limits are all nice and defined, we can combine them together. So, $\lim_{x\to 6}(f(x)\cdot g(x) - h(x)) = \left(\lim_{x\to 6} f(x)\right)\left(\lim_{x\to 6} g(x)\right) - \left(\lim_{x\to 6} h(x)\right)$. We are told these limits for f, g, and h are, respectively, 4, 9, and 6. So, the expression above $= 4\cdot 9 - 6 = 36 - 6 = 30$. There's not much to say here.

Please start working on the next assignment if you haven't.