213 Problem Set 4 Solutions

This is your last problem set before the exam (that should be obvious now). There were to be a total of ten questions here, five from each section. We returned to section 2.3. Make sure that the problems you chose were different OR work in 4.7. I will be doing all, because it's just better for all of you that way. Probably the 4.7 questions are more important for the exam than the ones here from 2.3.

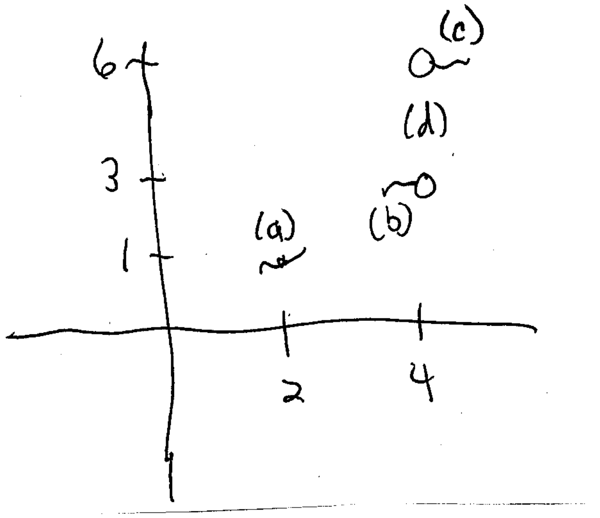
§2.3 47 For questions like this, and the next one, my way is to first mark the given information on the graph, and the connect the pieces. I could make fancy computer graphs, but I think for this kind of question, it's probably better to see hand-drawn graphs with the bits, like I would expect you would do. Here are the bits from the problem:

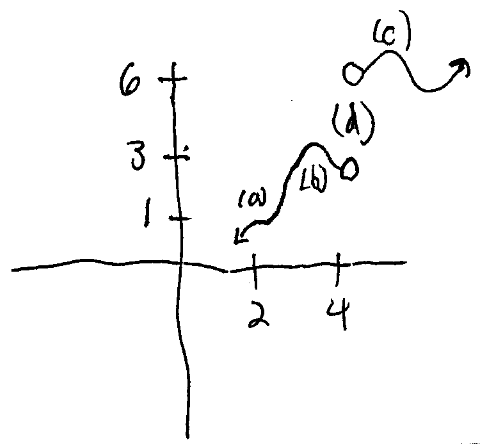
(a) $\lim_{x\to 2} f(x) = 1$ so we need to approach $y = 1$ from both sides at $x = 2$. We could have a hole or just thru the point $(2, 1)$. I drew the first.

(b) $\lim_{x\to 4^-} f(x) = 3$ from the left side at $x = 4$ the value moves to $y = 3$.

(c) $\lim_{x\to 4^+} f(x) = 6$ and from the right side at $x = 4$ the value moves to $y = 6$.

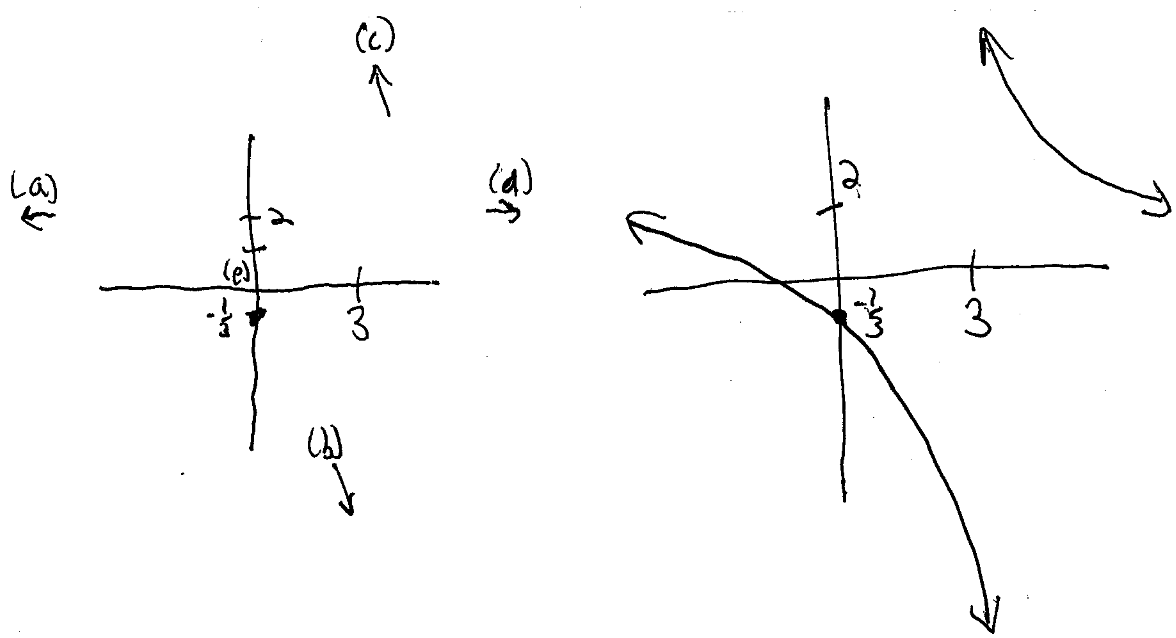
(d) and from $x = 4$ is not defined, so there is *no* value for $x = 4$. I have labeled the bits on my graph.





(next question on the next page)

§2.3 49 Same as above. Here are the bits:
(a) $\lim_{x\to-\infty} f(x) = 2$ so on the left of the graph go to $y = 2$.
(b) $\lim_{x\to 3^-} f(x) = -\infty$ so as we approach 3 from the left, go down off the bottom.
(c) $\lim_{x\to 3^+} f(x) = \infty$ and as we approach 3 from the right, go up off the top.
(d) $\lim_{x\to\infty} f(x) = 2$ like (a), this says that on the right of the graph go to $y = 2$.
(e) $f(0) = -\frac{1}{3}$ This one is just plot the point $(0, -\frac{1}{3})$.



As an unimportant detail, the function they might have in mind is $f(x) = 2 + \frac{7}{x-3}$, which satisfies all the given conditions. If you want you can make your own graph of that.

§2.5 8 Classify discontinuities for $f(t) = \frac{t+3}{t^2+5t+6}$. A good first move for all purposes is to factor the denominator. Yes, factoring is something we're doing a lot of, probably most of our algebra work so far it seems. $f(t) = \frac{t+3}{t^2+5t+6} = \frac{t+3}{(t+3)(t+2)}$. The only potential problems are going to come from the denominator equalling zero. $f(t)$ is undefined at both $t = -3$ and $t = -2$. So there's no hope of it being continuous at either of those points. Everywhere else is continuous. I would compute the value and the two side limits, but there are no values, so we only have the two side limits. In fact, the two sides for $t = -3$ are the same. Here's the work for that: first notice that $f(-3) = \frac{0}{0}$, so $\lim_{t\to-3} f(t) = \lim_{t\to-3} \frac{t+3}{t^2+5t+6} = \lim_{t\to-3} \frac{t+3}{(t+3)(t+2)} = \lim_{t\to-3} \frac{1}{t+2} = -1$. Because both sides are the same, the whole limit exists. This discontinuity is removable. We could include $f(-3) = -1$ and it would be continuous.

Now, what about $x = -2$? First notice that $f(-2) = \frac{1}{0}$. So now we only care about signs. $\lim_{t\to-2^-} f(t) = \lim_{t\to-2^-} \frac{t+3}{t^2+5t+6} = \lim_{t\to-2^-} \frac{t+3}{(t+3)(t+2)}$ for values near and less than -2 the numerator is positive, the first part of the denominator is also positive for the same reason, but the second is negative. We have $\frac{+}{+-}$. This is negative, so we have $\lim_{t\to-2^-} f(t) = -\infty$.

The other side is about the same, I'll just change a few bits: $\lim_{t\to-2^+} f(t) = \lim_{t\to-2^+} \frac{t+3}{t^2+5t+6} = \lim_{t\to-2^+} \frac{t+3}{(t+3)(t+2)}$ for values near and greater than -2 the numerator is positive, the first part of the denominator is also positive for the same reason, and the second is *still* positive this time. We have $\frac{+}{++}$. By the old saying "three positives make a positive" this is positive, so we have $\lim_{t\to-2^-} f(t) = \infty$. Clearly from the beginning, with all the infinite work, this is an infinite discontinuity; there's no fixing it.

§2.5 18 We want to make $f(x) = \begin{cases} e^{kx} & 0 \le x < 4 \\ x+3 & 4 \le x \le 8 \end{cases}$ continuous. The "given interval" part is a bit odd. The interval here is $[0, 8]$. There's no problem at the endpoints, despite it not being defined from the other side. Neither one gives a break in the graph. The only possible problem is at $x = 4$. We compute our three values: $f(4) = 4 + 3 = 7$ using the bottom rule, $\lim\limits_{x \to 4^-} f(x) = \lim\limits_{x \to 4^-} e^{kx} = e^{k4}$ using the top rule, and $\lim\limits_{x \to 4^+} f(x) = \lim\limits_{x \to 4^+} x + 3 = 7$ using the bottom rule. We want these three to be equal. We're glad that two already are. We want $7 = e^{k4} = 7$. So, we solve $7 = e^{k4}$ for k. We start by taking the logarithm of both sides (that's how we get variables out of exponents). I'm going to use the natural logarithm. I will show it for any other logarithm also. $\ln 7 = \ln(e^{k4}) = k4$, so $k = \frac{\ln 7}{4}$. If you use any other logarithm it looks similar, as follows: we solve $7 = e^{k4}$ for k. We start by taking the logarithm of both sides: $\log 7 = \log(e^{k4}) = k4(\log e)$, so $k = \frac{\log 7}{4 \log e}$. These are both the same number. Approximating it is not very important to me, but if you care, it is about 0.486478.

§4.7 12 $\lim\limits_{x \to \infty} \frac{2x-5}{4x}$. Compared to twice infinity, the -5 doesn't matter much so $\lim\limits_{x \to \infty} \frac{2x-5}{4x} = \lim\limits_{x \to \infty} \frac{2x}{4x} = \lim_{x \to \infty} \frac{2}{4} = \frac{1}{2}$. That wasn't bad.

§4.7 14 $\lim\limits_{x \to -\infty} \frac{3x^3-2x}{x^2+2x+8}$. The $3x^3$ term matters most in the numerator. The x^2 term matters most in the denominator. The rest are insignificant as we go back toward negative infinity, so $\lim\limits_{x \to -\infty} \frac{3x^3-2x}{x^2+2x+8} = \lim\limits_{x \to -\infty} \frac{3x^3}{x^2} = \lim\limits_{x \to -\infty} 3x = -3\infty = -\infty$.

Thankfully the §2.3 and §4.7 questions were both quite short. Please look at the ones you didn't do.