213 Problem Set 9 Solutions

4.4 30. Ok, this is a pretty standard problem for us now (I hope). $y = x^2 + \frac{2}{x}$. It's worth noting from the beginning - there's a problem at $x = 0$, but we are only considering $[1, 4]$. This equals $y = x^2 + 2x^{-1}$. The derivative is $y' = 2x - 2x^{-2}$. We can factor this: $y' = 2x^{-2}(x^3 - 1)$. We again have the $x = 0$ problem for the first factor, but we can ignore that. The second one factors as $y' = 2x^{-2}(x^3 - 1) = 2x^{-2}(x^2 - x + 1)(x - 1)$. There's a lot of stuff there, but mostly it doesn't concern us. The 2 can't be zero. The x^{-2} only causes problem at zero, where we're not looking. The $x^2 - x + 1$ *also* can't be zero (if you try to find roots, they are not real), which only leaves the $x - 1 = 0$ at the end, which means we should worry about $x = 1$, which we were already going to because it's an endpoint. So ... we check only $x = 1$ and $x = 4$. We go back to the original function, $y = x^2 + \frac{2}{x}$ and find $y(1) = 1 + 2 = 3$. We also find $y(4) = 16 + \frac{2}{4} = 16\frac{1}{2}$. The second is bigger so must be the maximum the first is the minimum. There are no local values.

4.4 41. This time we don't have endpoints. We start with $y = x^3 - 12x$ and take a derivative, again $y' = 3x^2 - 12$, and factor $y' = 3(x^2 - 4) = 3(x - 2)(x + 2)$. This equals zero at ± 2. So, our places to check are the endpoints and ± 2. So, we go back to the original and find $y(2) = 2^3 - 12(2) = 8 - 24 = -16$ and $y(-2) = (-2)^3 - 12(-2) = -8 + 24 = 16$. At a point in between these two values, my favourite is zero, we see that $y' = -12$ so it is decreasing between. This says that there is a local minimum at 2 and a local maximum at -2. What happens at the ends? Because of the x^3 part, as x goes to positive infinity, y does also. On the other side, as x goes to negative infinity, y does also. Therefore there are no absolute maxima or minima, since the function keeps growing endlessly to positive infinity and decreasing endlessly to negative infinity.