Here are some problems for us getting started in general topology:

From the book: §2 4,6

Is the collection $\mathcal{T}_\infty = \{U \mid X \backslash U \text{ is infinite or empty or all of } X\}$ a topology on X?

Show that the set of open (two-dimensional) rectangles forms a basis for a topology on $\mathbb{R}^2$. Show that the set of open disks also forms a basis for a topology on $\mathbb{R}^2$.

Find three topologies on the five point set $X = \{a, b, c, d, e\}$ such that the first topology is a subset of the second and the second is a subset of the third, without using either the discrete or indiscrete topologies. Find a topology on X that is neither a subset nor a superset of each of the first three that you found.

Let X be a set and assume $p \in X$. Show that the collection $\mathcal{P}$, consisting of $\emptyset, X$, and all subsets of X containing p, is a topology on X. Also show that the collection $\mathcal{E}$, consisting of $\emptyset, X$, and all subsets of X that exclude p, is a topology on X.

An arithmetic progression in $\mathbb{Z}$ is a set $A_{a,b} = \{a + nb \mid n \in \mathbb{Z}\}$ with $a, b \in \mathbb{Z}$ and $b \neq 0$. Prove that the collection of arithmetic progressions $\mathcal{A} = \{A_{a,b} \mid a, b \in \mathbb{Z}, b \neq 0\}$ is a basis for a topology on $\mathbb{Z}$.

Paraphrase on basis:

A collection of subsets $\mathcal{B}$ of a set X is a basis for a topology if it satisfies:

B1: For each $x \in X$, there exists a $U \in \mathcal{B}$ such that $x \in U$.

B2: For each $x \in X$, if $x \in U, V \in \mathcal{B}$, then there exists a $W \in \mathcal{B}$ such that $x \in W \subset U \cap V$.

We define a set U to be open in the topology generated by $\mathcal{B}$, (which we denote $\mathcal{T}_\mathcal{B}$) if for all $x \in U$, there exists a $V \in \mathcal{B}$ such that $x \in V \subset U$.

Prove: a set is in $\mathcal{T}_\mathcal{B}$ if and only if it is a union of elements of $\mathcal{B}$. I.e. $\mathcal{T}_\mathcal{B} = \{\cup_{U \in \mathcal{S}} U \mid \mathcal{S} \subset \mathcal{B}\}$

One part of §3 4 is to prove that this is a topology on X.

The other part of §3 4 is to prove $\mathcal{T}_\mathcal{B} = \mathcal{T}_{\mathcal{B}'}$ if and only if for every $x \in X$, if $x \in U \in \mathcal{B}$, then there exists a $V \in \mathcal{B}'$ such that $x \in V \subset U$, and for every $x \in X$, if $x \in V \in \mathcal{B}'$, then there exists a $U \in \mathcal{B}$ such that $x \in U \subset V$.