371 Problem Set 1 Comments

1. "Try to obscure". I think calling on a multiplicative inverse of 0 isn't quite obscure enough.

2. Needs a function from $\mathbb{Q}$ to L and to show that it is a bijection. This is what is means by the technical term "one-to-one correspondence". Alas, it seems this language is not well-known (I can agree with that). Therefore I have accepted solutions with only a 1-1 proof. No proof whatsoever is insufficient. Make sure that at some point you address the "through the point (a, b) part.

3. This begins with something like: Suppose $a + b = 0$ and $a + c = 0$, and then prove that $b = c$. This is better done using the associative and commutative properties from the expression $c+a+b$, but ok done with cancelation. Saying they are both equal to $-a$ *assumes* that inverses are unique.

4. Work in a finite field mod p. Use u and v as nonzero perfect squares. Because they have two square roots each, it shouldnt be difficult to select them to make the equation fail.

5. This is false. Try different quadratic equations mod 3. Find one that has no solution by merely directly checking the three options for solutions, 0, 1, and 2. Or find one where the discriminant is not a perfect square mod 3.

7. Assume the trichotomy property and rewrite it to the goal by applying it to the quantity $x - y$. Do *not* assume $x = y, x < y$ or $x > y$, this is what you are trying to prove.

8. Do not induct on the size of the set, tempting as it may be. If you do that, it only proves it is true for finite sets. Here is an idea I like. Suppose $S \neq \emptyset$ and S has no least element. Consider $T = \mathbb{N} \backslash S$. Prove that $T = \mathbb{N}$ by induction, therefore S is empty. The induction step is: Suppose $0 \ldots k \in T$ (this version uses strong induction, which may not be the PMI d'Angelo meant). Can $k+1 \in T$? If not, then $k+1$ is the least element in S. Since this is not allowed $k + 1 \in T$. Hypothesis loading (letting the induction hypothesis be that T contains all elements up to k) can resolve the need for strong induction.

This can also be done by proving it for finite sets (using induction) and then arguing that you can "cut off" at some finite point and only concern yourself with the lower finite part. It doesn't matter where you cut off, using nonempty to pick an element will suffice.

9. Some ideas in class, probably the best idea is showing that there are two disjoint inductive subsets if there is an $0 < x < 1 \in \mathbb{N}$.

10. The formula is clear here. This is only a question of can you write a careful induction proof. If you didn't, look back at your proofs class materials as to how to do so.

12. The goal is to use 11. Let $f(x) = A(x + B) - B = Ax + AB - B = Mx + C$. Clearly $M = A$. We then have $B(M - 1) = C$, so $B = \frac{C}{M-1}$ and we can write $f(x) = M(x+\frac{C}{M-1})-\frac{C}{M-1}$. From this without using induction we see from 11. that the composition of n f's is $M^n(x + \frac{C}{M-1}) - \frac{C}{M-1}$. This is the complete solution.

13. You should find something about the consequences of including infinity or infinitesimals.

14. Presentation

15. Make x, y, and $x + y$ vectors. Draw a triangle. Repeat for vectors a, b, and c, draw a different triangle.

16. This is only a matter of carefully changing inequalities and words. But, do be very careful.

21. Show $\frac{2\alpha}{n} < \frac{t-\alpha^2}{2}$ and $\frac{1}{n^2} < \frac{t-\alpha^2}{2}$ for some n using the Archimedean property. At least that's how I did it. Others started with $2\alpha + 1 < (t - \alpha^2)n$ and went from there. Presented in class.

22. Let $q = k - 1$. You must assign a value for q. once you do, then follow the steps.

23. Presentation.

24. I did this in class. I told you to not hand it in. Some ignored me. Unsurprisingly, they did not get credit.

25. This is a rotation. If you let $a = \cos(\theta), b = \sin(\theta)$ it appears in standard form.

26. This is merely the identity times a, which is a dilation if $a > 0$ and a dilation and half-turn rotation if $a < 0$.

27. We did this in class (twice).

28. We talked about some ideas for this. Perhaps unsurprisingly no one completed it for handing in.

29. Addition should be a standard "divides" proof. Multiplication is a little tricky. I think the 'right' idea there is $m_2 = ap + m_1$ and $n_2 = bp + n_2$ and then multiply m_2 and n_2.

30. Whenever you have a counterexample it needs to be tangible and specific. I hope you have used a particular precise counterexample. A hypothetical counterexample might as well be "suppose we have a counterexample."

31. This should be *almost* exactly the same as the proof for modular equivalence, except $(1 + t^2) \mid p(t)$ implies there is a *polynomial* $q(t)$ such that $(1 + t^2)q(t) = p(t)$. [This was an unpresented presentation problem.]

32. As discussed in class, the key is that there are zero divisors, $0 = (1 + t)(1 - t + t^2)$.

33. The question asks you to long divide $\sum_{k=0}^{d} c_k t^k$ by $t^2 + 1$ and find the remainder. The answer is $A + Bt = \sum_k = 0^{d/2}(-1)^n(c_{2k} + c_{2k+1}t)$.

34. This is sophisticated algebra like the sophisticated analysis questions. It's not so important to me that people know how to do it.

35. Using the division algorithm and dividing by $(x - x_0)$ we find that $p(x) = (x - x_0)q(x) + r(x)$, but the degree of $r(x)$ should be less than the degree of $(x - x_0)$, so $r(x)$ is actually a constant. Call it r. Then we have $p(x) = (x - x_0)q(x) + r$. To find r compute at x_0 to get $p(x_0) = (x_0 - x_0)q(x_0) + r = r$. Therefore we have, as desired, $p(x) = (x - x_0)q(x) + p(x_0)$.