371 Problem Set 3 Comments

1. Presentation.

2. Directly from geometry we see $\tan\left(\frac{\theta}{2}\right) = \frac{y}{x+1} = t$. Intriguingly it is the same as $\frac{\sin\theta}{1+\cos\theta}$.

3. Done in class.

4.5 Done in class

We skipped §3.3. No problems from that section will be given credit.

18. All this is is finding an inverse and checking that it agrees with the matrix above. I will be surprised if anyone has a problem with it.

18.5 For $A \neq 0$, we find a circle of centre $-\frac{\beta}{A}$ and radius $\sqrt{|\beta/A|^2 - C/A}$. For $A = 0$, we find a line through the point $-\frac{C}{2\beta}$ in direction $\overline{\beta}i$. Be careful about the conjugates in the line answers. There were some issues there.

19. Do this step-by-step. Move the circle left by 2, then contract by a factor of $\frac{1}{2}$, finally invert. This produces $f(z) = \frac{1}{\frac{1}{2}(z-2)} = \frac{2}{z-2}$. There are other sequences, but all produce this answer.

20. Presentation.

21. Done in class.

22. Justification is necessary.

23. Justification is necessary.

24. My in class discussion for why $\frac{1}{x}$ preserves circles and lines can be modified for $\overline{z}$. This is best done by considering the single equation representing circles and lines. The second part is by composition. It is not sophisticated, but does need to be addressed. It is essential that at some point you clearly say that any circle or line can be expressed as the set of points satisfying $\Phi(z) = 0$, and that anything in this form represents a circle or a line. The important part here is being clear with language. This was a problem for many.

25. The point here is that this can now be done without any use of l'Hospital or Bernoulli's help. This went fine.

26. Same as 25.

27. Presentation

28. The equator maps to the unit circle in the complex plane. In three dimensions, it maps to itself. This is because the line through the north pole that intersects a point on the equator meets the complex plane at that same point because the complex plane runs through the equator of the sphere. Some justification using the formula and $p_3 = 0$ is needed here.

29. The answer (which needs to be simplified in order to see the point of the question) is that if $p = (p_1, p_2, p_3)$, then $q = (p_1, -p_2, -p_3)$. In some sense the $z$-component reflection corresponds to the inside-out effects of inversion and the $y$-component reflection corresponds to the reflection over the real axis that is also an effect of inversion. The justification is checking that the projections of these two points are reciprocals.

30. Discussed in class.

31. Not completed.