371 Problem Set 4 Comments

Chapter 2 I'm not very interested in Chapter 2, so I'll give comments only when they interest me. I'm very not wanting this to turn into a real analysis class.

20. This should be a pretty simple induction problem using the triangle inequality. Be sure to be explicit about where the triangle inequality and induction hypotheses are used.

21. This amounts to a combinatorial proof that $\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$. The sum represents the number of games by round and the right expression is the number of teams that lose - all but one.

23. Show every three terms are a positive addition, and that once you get far enough it is bigger than 1 and won't be getting any smaller. Don't forget to show that it converges conditionally.

25. This is interesting, although not really complex-related. The idea is since it diverges in magnitude the positives and negatives both diverge. Therefore, if you take enough positives or negatives from the infinite end, you can always get as positive or negative as you want. (This reasoning was omitted from the solutions submitted.) So, alternate above/below the goal. This will converge to the goal because as we continue, the terms themselves go to zero.

26. No one attempted, but it is interesting. The idea is basically the same as 25, but now we have two dimensions. If all the terms are on a line, then the sum will stay on the line. If not, then it could be rearranged in all sorts of creative ways.

Chapter 4

4.1 Answers:

$$\sum_{n=0}^{\infty} nz = \left(\frac{1}{1-z}\right)^2 - \left(\frac{1}{1-z}\right)$$

$$\sum_{n=0}^{\infty} n^3 z = 6\left(\frac{1}{1-z}\right)^4 - 12\left(\frac{1}{1-z}\right)^3 + 7\left(\frac{1}{1-z}\right)^2 - \left(\frac{1}{1-z}\right)$$

$$\sum_{n=0}^{\infty} n^4 z = 24\left(\frac{1}{1-z}\right)^5 - 60\left(\frac{1}{1-z}\right)^4 + 50\left(\frac{1}{1-z}\right)^3 - 15\left(\frac{1}{1-z}\right)^2 + \left(\frac{1}{1-z}\right)$$

4.2 This question is really asking, given any p(n), find the associated q(w). It's probably not easy.

4.3 Presentation

4.4 $\sum nz^n$. Consider $|nz^n| = n|z^n| = n|z|^n = n$. By the divergence test, since the magnitude doesn't go to zero, the series cannot converge.

4.6 $e^z = e^{(z-z_0)+z_0} = e^{z-z_0}e^{z_0} = e^{z_0}\sum \frac{(z-z_0)^n}{n!}$. That's pretty much the entire solution.

4.7 $\frac{z}{z^4+9} = \frac{z}{9}\frac{1}{1-(-\frac{z^4}{9})} = \frac{z}{9}\sum(-1)^n\left(\frac{z^4}{9}\right)^n = \sum(-1)^n\frac{z^{4n+1}}{9^{n+1}}$. This converges when $\left|\frac{z^4}{9}\right| < 1$, i.e. when $|z| < \sqrt{3}$.

4.8 Merely use example 1.2.

4.9 $\sum \frac{z^n}{4^{n+2}} = \frac{1}{16}\sum\left(\frac{z}{4}\right)^n = \frac{1}{16}\frac{1}{1-z/4} = \frac{1}{16-4z}$. Again that's pretty much the entire solution.

4.10 The first is 1/e, with a nice limit in the middle. The second is 1.

4.11 Be sure to make some connections to Bernoulli numbers.

4.12 Seems straightforward. The second is a standard calc 2 problem.

4.13 $\frac{1}{z} = \frac{1}{p-(p-z)} = \frac{1}{p}\frac{1}{1-(p-z)/p} = \frac{1}{p}\sum\left(\frac{p-z}{p}\right)^n = \sum\left(\frac{(p-z)^n}{p^{n+1}}\right)$. This converges on the circle centred at $p$ of radius $|p|$, unsurprisingly.

4.14 $\frac{1}{1-z} = \frac{1/z}{1/z-1} = -\frac{1}{z}\frac{1}{1-1/z} = -\frac{1}{z}\sum z^{-n} = -\sum z^{-n-1}$. This is a geometric series that converges for $|1/z| < 1$, i.e. $|z| > 1$.

4.15 presentation

4.16 partial fractions splits into $\frac{1}{1-z} - \frac{1}{2-z}$ the first we can expand in negative powers as in 14, and the second in positive powers, giving the desired annulus of convergence. A curious idea.

4.17 The answer is $\frac{1}{z}^2 - 1$. The work probably involves differentiating the sum of the integral of the given series.