Number Theory 3: Unique Factorization

Objectives

⇒ To understand what is meant by *unique factorization into primes*

⇒ To continue to investigate the relationship between the number of factors an integer has and its representation in factored form

An integer with a number of factors can be represented in factored form in many different ways. For example, some of the representations of the number 24 are:

$2 \cdot 12 \qquad 3 \cdot 8 \quad 6 \cdot 2 \cdot 2 \qquad 2^3 \cdot 3$

Notice that in each of the first three forms, one of the factors is not prime. If we were to break down each of the composite factors into primes,

the result in each case would be $2^3 \cdot 3$. This is an example of the Fundamental Theorem of Arithmetic.

Fundamental Theorem of Arithmetic:

Every number can be written as a product of primes in one and only one way.

If a prime number appears as a factor more than once, we generally use exponential notation. Also, it is common to present the prime factors in ascending order. Thus instead of expressing 60 as

$2 \cdot 3 \cdot 2 \cdot 5$, we would write $2^2 \cdot 3 \cdot 5$.

Today you will begin to investigate how the factored form of an integer is related to the number of factors it has. It turns out that all integers with exactly three factors, for example, "look" pretty much the same when factored. There are only a few basic "forms" for numbers with exactly four factors. Keep these ideas in mind while doing the following activities.

Activity 1: Table of unique factorizations

Completing the table on the following page and looking for patterns in the results will help you make some conjectures about the composition of integers with certain numbers of factors. When listing factors, remember to include 1 and the integer itself.

Number	List of all Factors	Number of Factors	Prime Factorization
2	2 , 1	2	2
3			
4	1 , 2 , 4	3	2^2
5			
6			
7			
8			
9			
10			
11			
12	1 , 2 , 3 , 4 , 6 , 12	6	$2^2 \cdot 3$
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			

Refer to your table to answer the following questions:

1. What kinds of numbers have exactly two factors ?

2. What kinds of numbers have an odd number of factors ?

3. What is the general form of a number that has exactly three factors ?

4. Give an example of a number greater than 400 that has exactly three factors.

5. Can you predict the form of a number with exactly 5 factors ?

   Give two or three examples of numbers greater than 200 with exactly 5 factors.

6. Look at the number of factors that 2 , 4 , 8 , 16 and 32 have.

   How many factors would 128 have ?

   How many factors would 2^n have ?

   Without listing them, tell how many factors 81 has.

   In general, how many factors does p^n have, where p represents a prime number ?

7. You know already that an integer in the form p^3, where p represents a prime number, has exactly four factors.

Are there any integers that have exactly four factors that are not in this form ?

What are the possible forms for integers with exactly four factors ?