Number Theory 4: More about Prime Factorization Representation

Objectives

⇒ To make some formal conjectures about the form of integers with given numbers of factors

⇒ To investigate and make conjectures about more complex forms

Activity 1: Summary of yesterday's investigations

During the last class, you should have come up with conjectures about integers with:

⇒ exactly two factors

⇒ exactly three factors

⇒ an odd number of factors

⇒ exactly four factors .

In addition, you should have formulated a conjecture about the exact number of factors of integers of the form p^n where p is prime.

Summarize your findings, and be prepared to defend your conjectures to the class. Your instructor may ask several groups to present informal proofs of their results.

Activity 2: Integers of the form $p_1 p_2 p_3 p_4 \ldots p_n$

1. You have already found that integers of the form $p_1 p_2$ have exactly four factors. How many factors do integers of the form $p_1 p_2 p_3$ (where all of the p_i represent different prime numbers) have ? What about integers of the form $p_1 p_2 p_3 p_4$ and $p_1 p_2 p_3 p_4 p_5$?

HINT: a technique you learned in M118 will help you count the factors very efficiently.

Make a conjecture about the number of factors of an integer of the form $p_1 p_2 p_3 p_4 \ldots p_n$.

2. Find two different numbers with exactly 32 factors.