Number Theory 5: LCM and GCD

Objectives

- ⇒ To learn what is meant by the *least common multiple* (LCM) of a set of integers, and develop an algorithm for finding it
- ⇒ To learn what is meant by the *greatest common divisor* (GCD) of a set of integers, and develop an algorithm for finding it

Activity 1: Finding least common multiples

The *least common multiple* (LCM) of two integers x and y is the smallest integer that is a multiple of both x and y. In other words, it is the smallest number that both x and y will divide. We denote the least common multiple of x and y LCM (x , y). Similarly, the LCM of a set of integers is the smallest number that is divisible by all of the integers in the set. One of your tasks today will be to come up with a procedure for finding the LCM of two or more integers.

1. Find the LCM of:

   a. 4 and 9

   b. 4 and 14

   c. 6 and 25

   d. 6 and 21

   e. 8 and 15

   f. 8 and 12

   g. $3 \cdot 5$ and $2 \cdot 7$

   h. $3 \cdot 5$ and $3 \cdot 7$

   i. $p_1 p_2$ and $p_3 p_4$

   j. $p_1 p_2$ and $p_1 p_3$

2. a. Under what circumstances is the LCM of two numbers equal to their product ?

b. Will the LCM of two numbers ever be greater than their product ?

c. Give an example of two numbers for which the LCM is less than their product.

d. Find the least common multiple of $5 \cdot 7$, $2 \cdot 11$, and 3^4.

3. Find the LCM of:

a. 5 and 20

b. 16 and 48

c. 30 and 6

d. 5 and 1

e. p_1 and $p_1 p_2 p_3$

f. p_1 and $p_1 p_2{}^3$

g. 9 , 36 and 12

h. 10, 100 and 1000

4. Write a general statement about the LCM of two integers if one of them is a multiple of the other.

5. Complete the following table.

<u>**numbers**</u>	<u>**numbers in factored form**</u>	<u>**factored form of LCM**</u>	<u>**LCM**</u>
4 , 14	$2 \cdot 2$, $2 \cdot 7$	$2 \cdot 2 \cdot 7$	28
6 , 21			
8 , 12	2^3, $2^2 \cdot 3$	$2^3 \cdot 3$	24
10 , 16			
56 , 24			
36 , 120			
	$7^2 \cdot 5$, $2^2 \cdot 5^3 \cdot 7$		
	$p_1 p_2^{\,3} p_3$, $p_1^{\,4} p_2 p_3$		
18 , 21 , 35			
24 , 22 , 27			

6. Write an explanation of how to find the LCM of two (or more) integers. The first step in your explanation should be "Factor each integer into powers of primes"

Activity 2: Finding greatest common divisors

The *greatest common divisor* (GCD) of two integers x and y is the largest integer that divides both x and y . We denote the greatest common divisor of x and y GCD (x , y). Similarly, the GCD of a set of integers is the largest number that divides all of the integers in the set. Your second task today will be to come up with a procedure for finding the GCD of two or more integers.

1. Find the GCD of:

   a. 1 and 14

   b. 15 and 16

   c. 8 and 25

   d. 24 and 49

   e. p_1 and p_2 where p_1 and p_2 represent prime numbers.

2. Two integers that have no common factor (other than 1) are called *relatively prime* . Write a statement about the GCD of two integers that are relatively prime.

3. Complete the table below.

numbers	numbers in factored form	factored form of GCD	GCD
6 , 9	$2 \cdot 3$, $3 \cdot 3$	3	3
24 , 42	$2^3 \cdot 3$, $2 \cdot 3 \cdot 7$	$2 \cdot 3$	6
16 , 96			
50 , 75			
252 , 210			
	$3^3 \cdot 5^4$, $2 \cdot 3 \cdot 5^3$		
	$p_1^{2} p_2^{3}$, $p_1^{3} p_2^{2}$		
6 , 15 , 21			
24 , 22 , 27			

4. Write a general procedure for finding the GCD of two or more integers. The first step in the procedure should be "Factor each integer into powers of primes."