Number Theory 6: Exploring the Relationship Between LCM and GCD

Objectives

⇒ To explore the relationship between the LCM and the GCD of a set of integers.

⇒ To develop a second algorithm for finding the LCM of two numbers.

Activity 1: General procedures for finding LCM and GCD

Your instructor will ask several groups of students to present their procedures for finding least common multiples and greatest common divisors. The class should discuss and revise these procedures and come up with statements that are suitable not only for T104 students, but also for use with children in the upper elementary grades. Your explanations of the procedures should be clear and explicit enough for a student to follow even if he or she had never found a LCM or a GCD before.

Activity 2: The relationship between the LCM and the GCD

Most students find it easier to determine the greatest common divisor of two integers than to find their least common multiple. There is a way to find the LCM of two numbers by first computing their GCD, and then using the relationship between LCD and GCM. Your task is to find that relationship and then use it to come up with an alternate method for finding least common multiples.

1. Suppose that you are given two numbers that share no common factors. How do you find their LCM ?

2. If GCD (x , y) = 1, then LCM (x , y) = _____ .

3. In questions one and two you found that when two integers have GCD equal to 1, then their LCM is the *product* of the numbers. The following table will help you find a relationship between the product of two numbers and their LCM when their GCD is not equal to 1.
Complete the following table. To see what is happening, you must use the factored form of all numbers in your table.

numbers	**factored form**	**product**	**GCD**	**LCM**
6 and 21	$2 \cdot 3$, $3 \cdot 7$	$2 \cdot 3 \cdot 3 \cdot 7$	3	$2 \cdot 3 \cdot 7$
10 and 35				
70 and 105				
4 and 32	2^2, 2^5	$2^2 \cdot 2^5$	2^2	2^5
9 and 81				
45 and 54	$3^2 \cdot 5$, $2 \cdot 3^3$	$3^2 \cdot 5 \cdot 2 \cdot 3^3$	3^2	$2 \cdot 3^3 \cdot 5$
100 and 24				
72 and 40				

4. a. Suppose that you know that the product of two integers is 360 and their GCD is 1 . What is their LCM ?

b. Suppose that you know that the product of two integers is 360 and their GCD is 6 . What is their LCM ?

5. Make a conjecture about the relationship between the product of two integers, their LCM and their GCD. Test it with several examples. Does your conjecture still hold if the GCD of the two integers is 1 ?

6. Can you generalize your conjecture to finding the LCM of three or more numbers ? If yes, explain why. If no, find a counterexample.