

Rational Numbers 10: Conversions between Fractions and Decimals

Reminder definitions:

A *rational number* is any number that *can be written* as a fraction.

A *real number* is any number that can be written as decimal.

Activity 1: Converting fractions to decimal form

1. Convert each of the following fractions to decimal form by hand. Please pay careful attention to your steps.

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{1}{4}$

d. $\frac{1}{5}$

e. $\frac{1}{6}$

f. $\frac{1}{7}$

g. $\frac{1}{8}$

h. $\frac{1}{9}$

i. $\frac{1}{10}$

j. $\frac{1}{11}$

k. $\frac{1}{15}$

l. $\frac{1}{16}$

2. Classify the decimals above into two groups.

3. Suppose you are given a positive number d . How can you tell by looking at d or information about d , if the decimal representation of $1/d$ will terminate without dividing?

4. If $1/d$ is a repeating decimal, will $2/d$, $3/d$, $4/d$, and so on also be repeating, or does whether n/d is repeating depend more on n and d ? How? In other words, if $1/d$ is repeating, is it possible that n/d is not repeating?

5. Suppose that you are given a fraction n/d , and you begin to divide in order to convert it into decimal form. Is it possible that your division will go on forever without terminating or repeating? (Hints: think carefully about your work in converting by hand. What happened in your work to first tell you that the division terminated? What is the similar first sign that it repeated? Is it possible that *neither* of these happen? Why or why not?)

Activity 2: Converting terminating decimals to fraction form

1. For justifying multiplication and division we often converted terminating decimals into fractions. Write equivalent fractions to these decimals.

a. 0.345

b. 4.12

c. -1.2359

2. Describe how to convert any terminating decimal into fraction form.

Activity 3: Converting repeating decimals to fraction form

There are several methods for converting repeating decimals into fraction form. The one that is most accessible to students in the upper elementary grades is illustrated below:

Example: Convert $0.\overline{09}$ to fraction form.

Solution: We want to find a fraction $F = 0.09090909 \dots = 0.\overline{09}$.

Then $100F = 09.090909 \dots = 9.\overline{09}$.

We have two equations:

$$100F = 9.090909 \dots = 9.\overline{09}$$

$$F = 0.09090909 \dots = 0.\overline{09}$$

If we subtract the second from the first we get:

$$99F = 9 \text{ or } F = 9/99$$

1. a. Review the example carefully. Why did we decide to multiply each side of the original equation by 100?

b. What was the purpose of subtracting the two equations?

2. Use the above method to convert each of the following into fractions:

a. $0.\overline{3}$

2. Use the method of this activity to convert each of the following into fractions:

b. $4.\overline{452}$

c. $1.5\overline{32}$ (Hint: you will need to multiply twice.)

3. Is it possible to use the method of this activity to convert the following decimals into fractions?

a. $0.121121112 \dots$

b. $1.234567891011121314 \dots$

4. Are the numbers in question 3. equivalent to any fractions? Are they real numbers? What type of numbers are they? List five other numbers like these.