Chapter IV
Arabic Primacy with Hindu, Chinese, and Maya Contributions

Bhāskara II (also known as Bhāskarāchārya or Bhāskara the Learned; 1115–c. 1185)

Bhāskara II was the leading Indian astronomer and mathematician of the 12th century. A son of the Brahman Maheśvara, he was probably born in Vijayapura (now Bijāpur in Mysore). He served as head of the astronomical observatory at Ujjain (the chief mathematical center of ancient and medieval India). He was also a noted astrologer. According to tradition, his astrological meddling and an unfortunate twist of fate deprived his daughter of her only chance for marriage and happiness. Perhaps to console her, he named his first book *Līlāvatī* after her.

Bhāskara wrote at least two books on mathematics. The first—*Līlāvatī* ("The Beautiful")—has thirteen chapters on arithmetic, geometry, and algebra. The second—*Bījagaṇita* ("Seed Counting")—contains twelve chapters on algebra. In these books, Bhāskara skillfully used the decimal system. In the decimal system he treated zero as a complete number and not simply as a positional notation. Like earlier Indian mathematicians he discussed arithmetic operations with zero. He went beyond them, however, in examining the universal problem of division by zero. Although he stated that a quantity divided by zero gives an infinite quantity (for example, in modern symbols $\frac{3}{0} = \infty$), and that $\frac{a}{0} \times 0 = a$, he knew that the reverse process of multiplication by zero did not produce a unique value. His inclusion of negative numbers in computations and recognition that the square roots of numbers have a positive and a negative value helped lead to the subsequent acceptance of negative numbers, even though he noted that "people do not approve of negative solutions." He grasped the convention of signs (minus times minus equals plus, and plus times minus equals minus).

In algebra, Bhāskara improved upon the work of the Indian mathematician Brahmagupta (598–c. 665). He introduced some symbolic notation into his own basically rhetorical problems by representing negative quantities by a superior dot over positive quantities and by representing unknown quantities by the initial syllables of the words for colors. He solved first and second degree indeterminate equations and reduced quadratic equations to a single type to solve them. Like many later mathematicians Bhāskara was intrigued by what is now known as Pell's equation (in modern symbols, $ax^2 + 1 = y^2$) and provided a solution. In the case when $a = 8$, $x = 6$ and $y = 17$ satisfy the equation. In geometry he studied regular polygons with as many as 384 sides and obtained an approximation of π as 3.14166.

Bhāskara wrote two books on mathematical astronomy—*Siddhāntaśiromani* ("Head Jewel of Accuracy," 1150) and *Karaṇakutūhala* ("Calculation of Astronomical Wonders," 1183). These texts cover astronomical observations, lunar and solar eclipses, planetary positions and conjunctions, heliacal risings and settings, cosmography, geography, spherical trigonometry, astronomical equipment, and mathematical techniques used in astronomy.

49. From *Līlāvatī**

(Arithmetic and Geometry)

BHĀSKARA II

CHAPTER III, Section VI
RULE OF PROPORTION

74. *Rule of three inverse.*[1]

If the fruit diminish as the requisition increases, or augment as that decreases, they, who are skilled in accounts, consider the rule of three terms to be inverted.[2]

When there is diminution of fruit, if there be increase of requisition, and increase of fruit if there be diminution of requisition, then the inverse rule of three is [employed]. For instance,

75. When the value of living beings[3] is regulated by their age; and in the case of gold, where the weight and touch are compared; or when heaps[4] are subdivided; let the inverted rule of three terms be [used].

76. *Example.* If a female slave sixteen years of age, bring thirty-two [*nishcas*], what will one aged twenty cost? If an ox, which has been worked a second year, sell for four *nishcas* what will one, which has been worked six years, cost?

1st Qu. Statement: 16 32 20. Answer: 25-3/5 *nishcas.*

2d Qu. Statement: 2 4 6. Answer: 1-1/3 *nishca.*

77. *Example.* If a *gadyánaca* of gold of the touch of ten may be had for one *nishca* [of silver], what weight of gold of fifteen touch may be bought for the same price?

Statement: 10 1 15. Answer 2/3.

78. *Example.* A heap of grain having been meted with a measure containing seven *ad'hacas,* if a hundred such measures were found, what would be the result with one containing five *ad'hacas?*

Statement: 7 100 5. Answer 140.

80. *Example.* If the interest of a hundred for a month be five, say what is the interest of sixteen for a year? Find likewise the time from the principal and interest; and knowing the time and produce, tell the principal sum.

Statement:
1 12
100 16
5

Answer: the interest is 9-3/5.

To find the time; Statement:
1
100 16
5 48/5

Answer: months 12.

*Source: The selections from *Līlāvatī* (49) and *Bījagṇita* (50) are taken from the English translation from the Sanskrit by Henry Thomas Colebrooke in 1817. They appeared in Henrietta Midonick, *The Treasury of Mathematics,* 117-140.

To find the principal; Statement: 1 12
100
5 $^{48}/_{5}$

Answer: principal 16.

81. *Example.* If the interest of a hundred for a month and one-third, be five and one fifth, say what is the interest of sixty-two and a half for three months and one fifth?

Statement: $\frac{4}{3}$ $\frac{16}{5}$ / $\frac{100}{1}$ $\frac{125}{2}$ / $\frac{26}{5}$

Answer: interest 7-$^{4}/_{5}$.

82. *Example of the rule of seven:* If eight, best, variegated, silk scarfs, measuring three cubits in breadth and eight in length, cost a hundred [*nishcas*]; say quickly, merchant, if thou understand trade, what a like scarf, three and a half cubits long and half a cubit wide, will cost.

Statement: 3 $^{1}/_{2}$
8 $^{7}/_{2}$
8 1
100

Answer: *Nishca* 0, *drammas* 14, *panas* 9, *cacini* 1, *cowryshells* 6-$^{2}/_{3}$.

83. *Example of the rule of nine:* If thirty benches, twelve fingers thick, square of four wide, and fourteen cubits long, cost a hundred [*nishcas*]; tell me, my friend, what price will fourteen benches fetch, which are four less in every dimension?

Statement: 12 8
16 12
14 10
30 14
100

Answer: *Nishcas* 16-$^{2}/_{3}$.

CHAPTER IV, SECTION VI PERMUTATION AND COMBINATION

114. *Example:* In a pleasant, spacious and elegant edifice, with eight doors,[5] constructed by a skilful architect, as a palace for the lord of the land, tell me the permutations of apertures taken one, two, three, &c.[6] Say, mathematician, how many are the combinations in one composition, with ingredients of six different tastes, sweet, pungent, astringent, sour, salt and bitter,[7] taking them by ones, twos, or threes, &c.

Statement [1st Example]: 8 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8.

Answer: the number of ways in which the doors may be opened by ones, twos, or threes, &c. is 8, 28, 56, 70, 56, 28, 8,
1 2 3 4 5 6 7
1. And the changes on the apertures of
8
the octagon palace amount to 255.[8]

Statement 2d example: 6 5 4 3 2 1
1 2 3 4 5 6.

Answer: the number of various preparations with ingredients of divers tastes is 6, 15, 20, 15, 6, 1.[9]
1 2 3 4 5 6

CHAPTER V, PROGRESSIONS SECTION I, ARITHMETICAL PROGRESSION

115. *Rule:* Half the period, multiplied by the period added to unity, is the sum of the arithmeticals one, &c. and is named their addition. This, being multiplied by the period added to two, and being divided by three, is the aggregate of the additions.

123. *Rule:*[10] half a stanza. The sum divided by the period, and the first term subtracted from the quotient, the remainder, divided by half of one less than the number of terms, will be the common difference.

124. *Example:* On an expedition to seize his enemy's elephants, a king marched two *yojánas* the first day. Say, intelligent calculator, with what increasing rate of daily march did he proceed, since he reached his foe's city, a distance of eighty *yojánas,* in a week?

Statement: First term 2; Com. diff.? Period 7; Sum 80.

Answer: Com. diff. $^{22}/_{7}$.

125. *Rule:*[11] From the sum of the progression multiplied by twice the common increase, and added to the square of the difference between the first term and half that increase, the square root being extracted, this root less the first term and added to the [above-mentioned] portion of the increase, being divided by the increase, is pronounced to be the period.

126. *Example:* A person gave three *drammas* on the first day, and continued to distribute alms increasing by two [a day]; and he thus bestowed on the priests three hundred and sixty *drammas:* say quickly in how many days?

Statement: First term 3; Com. diff. 2; Period? Sum. 360.

Answer: Period 18.

SECTION II GEOMETRICAL PROGRESSION

127. *Rule:* a couplet and a half. The period being an uneven number, subtract one, and note "multiplicator;" being an even one, halve it, and note "square:" until the period be exhausted. Then the produce arising from multiplication and squaring [of the common multiplier] in the inverse order from the last, being lessened by one, the remainder divided by the common multiplier less one, and multiplied by the initial quantity, will be the sum of a progression increasing by a common multiplier.

128. *Example:* A person gave a mendicant a couple of cowry shells first; and promised a two-fold increase of the alms daily. How many *nishcas* does he give in a month?

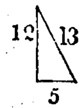
Statement: First term, 2; Two-fold increase, 2; Period, 30.

Answer, 2147483646 cowries; or 104857 *nishcas*, 9 *drammas*, 9 *panas*, 2 *cacinis*, and 6 shells.

CHAPTER VI PLANE FIGURE

158. *Example.* Where the difference of the side and upright is seven and hypotenuse is thirteen, say quickly, eminent mathematician, what are the side and upright?[12]

Statement. Difference of side and upright 7. Hypotenuse 13. Proceeding as directed, the side and upright come out 5 and 12. See

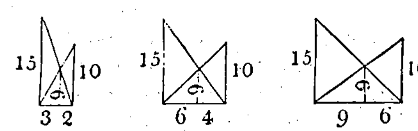


159. *Rule.*[13] The product of two erect bambus being divided by their sum, the quotient is the perpendicular from the junction [intersection] of threads passing reciprocally from the root [of one] to the tip [of the other.] The two bambus, multiplied by an assumed base, and divided by their sum, are the portions of the base on the respective sides of the perpendicular.

160. *Example.* Tell the perpendicular drawn from the intersection of strings stretched mutually from the roots to the summits of two bambus fifteen and ten cubits high standing upon ground of unknown extent.

Statement Bambus 15, 10. The perpendicular is found 6.

Next to find the segments of the base: let the ground be assumed 5; the segments come out 3 and 2. Or putting 10, they are 6 and 4. Or taking 15, they are 9 and 6. See the figures



In every instance the perpendicular is the same: viz. 6.

The proof is in every case by the rule of three: if with a side equal to the base, the bambu be the upright, then with the segment of the base what will be the upright?[14]

161. *Aphorism.*[15] That figure, though rectilinear, of which sides are proposed by some presumptuous person, wherein one side[16] exceeds or equals the sum of

the other sides, may be known to be no figure.

162. *Example:* Where sides are proposed two, three, six, and twelve in a quadrilateral, or three, six and nine in a triangle, by some presumptuous dunce, know it to be no figure.

CHAPTER XII
PULVERIZER[17]

248–252. *Rule.* In the first place, as preparatory to the investigation of a pulverizer,[18] the dividend, divisor and additive quantity[19] are, if practicable, to be reduced by some number.[20] If the number, by which the dividend and divisor are both measured, do not also measure the additive quantity, the question is an ill put [or impossible] one.

249–251. The last remainder, when the dividend and divisor are mutually divided, is their common measure.[21] Being divided by that common measure, they are termed reduced quantities.[22] Divide mutually the reduced dividend and divisor, until unity be the remainder in the dividend. Place the quotients one under the other; and the additive quantity beneath them, and cipher at the bottom.[23] By the penult multiply the number next above it and add the lowest term. Then reject the last and repeat the operation until a pair of numbers be left. The uppermost of these being abraded[24] by the reduced dividend, the remainder is the quotient. The other [or lowermost] being in like manner abraded by the reduced divisor, the remainder is the multiplier.

252. Thus precisely is the operation when the quotients are an even number.[25] But, if they be odd, the numbers as found must be subtracted from their respective abraders, the residues will be the true quotient and multiplier.

253. *Example.* Say quickly, mathematician, what is that multiplier, by which two hundred and twenty-one being multiplied, and sixty-five added to the product, the sum divided by a hundred and ninety-five becomes exhausted?

Statement: Dividend 221 Additive 65.
Divisor 195

Here the dividend and divisor being mutually divided, the last of the remainders (or divisors) is 13. By this common measure, the dividend, divisor and additive, being reduced to their least terms, are Divd. 17 Addve. 5.
Divr. 15

The reduced dividend and divisor being divided reciprocally, and the quotients put one under the other, the additive under them, and cipher at the bottom, the series which results is 1
7
5
0

Then multiplying by the penult the number above it and proceeding as directed, the two quantities are obtained 40
35

These being abraded by the reduced dividend and divisor 17 and 15, the quotient and multiplier are obtained 6 and 5. Or, by the subsequent rule (§262), adding them to their abraders multiplied by an assumed number, the quotient and multiplier [putting 1] are 23 and 20; or, putting 2, they are 40 and 35: and so forth.[26]

CHAPTER XIII

277. Joy and happiness is indeed ever increasing in this world for those who have *Līlāvatī* clasped to their throats, decorated as the members are with neat reduction of fractions, multiplication and involution, pure and perfect as are the solutions, and tasteful as is the speech which is exemplified.

NOTES

1. *Vyasta-trairasica* or *Viloma-trairasica,* rule of three terms inverse.

2. The method of performing the inverse rule has been already taught (§70). "In the inverse method, the operation is reversed." That is, the fruit to be multiplied by the argument and divided by the demand.

When fruit increases or decreases, as the demand is augmented or diminished, the direct rule *(crama-trairasica)* is used. Else the inverse.

3. Slaves and cattle. The price of the older is less; of the younger, greater.

4. When heaps of grain, which had been meted with a small measure, are again meted with a larger one, the number decreases; and when those, which had been meted with a large measure, are again meted with a smaller one, there is increase of number.

5. *Muc'ha,* aperture for the admission of air: a door or window; . . . a portico or terrace.

6. The variations of one window or portico open (or terrace unroofed) and the rest closed; two open, and the rest shut; and so forth.

7. *Amera-cosha* 1.3.18.

8. An octagon building, with eight doors (or windows; porticos or terraces;) facing the eight cardinal points of the horizon, is meant.

9. Total number of possible combinations, 63.

10. The first term, period and sum being known, to find the common difference which is unknown.

11. The first term, common difference and sum being known, to find the period which is unknown.

12. This example of a case where the difference of the sides is given, is omitted by Suryadasa, but noticed by Ganesa. Copies of the text vary; some containing and others omitting, the instance.

13. Having taught fully the method of finding the sides in a right-angled triangle, the author next propounds a special problem. To find the perpendicular, the base being unknown.

14. On each side of the perpendicular, are segments of the base relative to the greater and smaller bambus, and larger or less analogously to them. Hence this proportion. "If with the sum of the bambus, this sum of the segments equal to the entire base be obtained, then, with the smaller bambu, what is had?" The answer gives the segment, which is relative to the least bambu. Again: "if with a side equal to the whole base, the higher bambu be the upright, then with a side equal to the segment found as above, what is had?" The answer gives the perpendicular let fall from the intersection of the threads. Here a multiplicator and a divisor equal to the entire base are both cancelled as equal and contrary: and there remain the product of the two bambus for numerator and their sum for denominator. Hence the rule.

15. The aphorism explains the nature of impossible figures. . . . In a triangle or other plane rectilinear figure, one side is always less than the sum of the rest. If equal, the perpendicular is nought, and there is no complete figure. If greater, the sides do not meet.

16. The principal or greatest side.

17. *Cuttaca-vyavahara* or *cuttacad'hyaya* determination of a grinding or pulverizing multiplier, or quantity such, that a given number being multiplied by it, and the product added to a given quantity, the sum (or, if the additive be negative, the difference) may be divisible by a given divisor without remainder.

In Brahmagupta's work the whole of algebra is comprised under this title of *Cuttacad'hyaya,* chapter on the pulverizer.

18. *Cuttaca* or *Cutta,* from *cutt,* to grind or pulverize; (to multiply: all verbs importing tendency to destruction also signifying multiplication.

The term is here employed in a sense independent of its etymology to signify a multiplier such, that a given dividend being multiplied by it, and a given quantity added to (or subtracted from) the product, the sum (or difference) may be measured by a given divisor.

The derivative import is, however, retained in the present version to distinguish this from multiplier in general; *cuttaca* being restricted to the particular multiplier of the problem in question.

19. *Cshepa,* or *cshepaca,* or *yuti,* additive. From *cship* to cast or throw in, and from *yu* to mix. A quantity superinduced, being either affirmative or negative, and consequently in some examples an additive, in others a subtractive, term.

20. *Visudd'hi,* subtractive quantity, contradistinguished from *cshepa* additive, when this is restricted to an affirmative one.

21. *Apavartana,* abridgment; abbreviation. Depression or reduction to least terms; division without remainder: also the number which serves to divide without residue; the common measure, or common divisor of equal division.

22. *Drid'ha,* firm: reduced by the common divisor to the least term.
23. *Tashta,* abraded; from *tacsh,* to pare or abrade: divided, but the residue taken, disregarding the quotient: reduced to a residue. As it were a residue after repeated subtractions.

Tacshana, the abrader; the divisor employed in such operation.
24. *P'hala-valli,* the series of quotients; to be reduced by the operation forthwith directed to only two terms.
25. Even, as 2, 4, 6, &c.
26. Putting 3, they are 57 and 50.

50. From *Bījagṇita*

(Algebra)

BHĀSKARA II

CHAPTER I, SECTION II
LOGISTICS OF NEGATIVE AND AFFIRMATIVE QUANTITIES

ADDITION

3. *Rule for addition of affirmative and negative quantities: half a stanza.* In the addition of two negative or two affirmative[1] quantities, the sum must be taken: but the difference of an affirmative and a negative quantity is their addition.

4. *Example.* Tell quickly the result of the numbers three and four, negative or affirmative, taken together: that is, affirmative and negative, or both negative or both affirmative, as separate instances: if thou know the addition of affirmative and negative quantities.

The characters, denoting the quantities known and unknown,[2] should be first written to indicate them generally; and those, which become negative, should be then marked with a dot over them.

Statement: 3.4. Adding them, the sum is found 7.
Statement: $\dot{3}.\dot{4}$. Adding them, the sum is $\dot{7}$.
Statement: 3.$\dot{4}$. Taking the difference, the result of addition comes out $\dot{1}$.
Statement: $\dot{3}$.4. Taking the difference, the result of addition is 1.

So in other instances, and in fractions likewise.

SUBTRACTION

5. *Rule for subtraction of positive and negative quantities: half a stanza.* The quantity to be subtracted being affirmative, becomes negative; or, being negative, becomes affirmative: and the addition of the quantities is then made as above directed.

6. *Example: half a stanza.* Subtracting two from three, affirmative from affirmative, and negative from negative, or the contrary, tell me quickly the result.

Statement: 3.2. The subtrahend, being affirmative, becomes negative; and the result is 1.
Statement: $\dot{3}.\dot{2}$. The negative subtrahend becomes affirmative; and the result is $\dot{1}$.
Statement: 3.$\dot{2}$. The negative subtrahend becomes affirmative; and the result is 5.
Statement: $\dot{3}$.2. The affirmative subtrahend becomes negative; and the result is $\dot{5}$.

MULTIPLICATION

7. *Rule for multiplication [and division] of positive and negative quantities: half a stanza.* The product of two quantities both affirmative, is positive.[3] When a positive quantity and a negative one are multiplied together, the product is negative. The same is the case in division.

SECTION III
CIPHER

12. *Rule for addition and subtraction of cipher: part of a stanza.* In the addition of cipher, or subtraction of it, the quantity, positive or negative, remains the same. But, subtracted from cipher, it is reversed.

13. *Example:* half a stanza. Say what is the number three, positive, or [the same number] negative, or cipher, added to cipher, or subtracted from it?

Statement: 3.$\dot{3}$.0. These, having cipher added to, or subtracted from, them, remain unchanged: 3.$\dot{3}$.0.
Statement: 3.$\dot{3}$.0. Subtracted from cipher, they become $\dot{3}$.3.0.

14. *Rule:* (completing the stanza, §12.) In the multiplication and the rest of the operations[4] of cipher, the product is cipher; and so it is in multiplication by cipher: but a quantity, divided by cipher, becomes a fraction the denominator of which is cipher.[5]

15. *Example: half a stanza.* Tell me the product of cipher multiplied by two;[6] and the quotient of it divided by three, and of three divided by cipher; and the square of nought; and its root.

Statement: Multiplicator 2. Multiplicand 0. Product 0.
[Statement: Multiplicator 0. Multiplicand 2. Product 0.]
Statement: Dividend 0. Divisor 3. Quotient 0.
Statement: Dividend 3. Divisor 0. Quotient the fraction $^3/_0$.

This fraction, of which the denominator is cipher, is termed an infinite quantity.[7]

16. In this quantity consisting of that which has cipher for its divisor, there is no alteration, though many be inserted or extracted; as no change takes place in the infinite and immutable God, at the period of the destruction or creation of worlds, though numerous orders of beings are absorbed or put forth.

Statement: 0. Its square 0. Its root 0.

SECTION IV
ARITHMETICAL OPERATIONS ON UNKNOWN QUANTITIES

17. "So much as" and the colours "black, blue, yellow and red,"[8] and others besides these, have been selected by venerable teachers for names of values[9] of unknown quantities, for the purpose of reckoning therewith.

18. *Rule for addition and subtraction:* Among quantities so designated, the sum or difference of two or more which are alike must be taken: but such as are unlike, are to be separately set forth.

19. *Example.* Say quickly, friend, what will affirmative one unknown with one absolute, and affirmative pair unknown less eight absolute, make, if addition of the two sets take place? and what will they make, if the sum be taken inverting the affirmative and negative signs?

Statement: *ya* 1 *ru* 1
ya 3 *ru* $\dot{7}$. Answer:
the sum is *ya* 2 *ru* $\dot{8}$ the known quantities. Hence the one side being divided by the residue of the first (letter or) colour, a value of the (letter or) colour which furnishes the divisor is obtained. If there be many such sides, by so treating those that constitute equations, by pairs, other values are found.

CHAPTER VII
VARIETIES OF QUADRATICS

178. *Example from ancient authors:* The square of the sum of two numbers, added to the cube of their sum, is equal to twice the sum of their cubes. Tell the numbers, mathematician!

The quantities are to be so put by the intelligent algebraist, as that the solution may not run into length. They are accordingly put *ya* 1 *ca* $\dot{1}$ and *ya* 1 *ca* 1. Their sum is *ya* 2. Its square *ya v* 4. Its cube *ya gh* 8. The square of the sum added to the cube is *ya gh* 8 *ya v* 4. The cubes of the two quantities respectively are *ya gh* 1 *ya v. ca bh* $\dot{3}$ *ca v. ya bh* 3 *ca gh* $\dot{1}$ cube of the first; and *ya gh* 1 *ya v. ca bh* 3 *ca v. ya bh* 3 *ca gh* 1 cube of the second; and the sum of these is *ya gh* 2 *ca v. ya bh* 6; and doubled, *ya gh* 4 *ca v. ya bh* 12. Statement for equal subtraction: *ya gh* 8 *ya v* 4 *ca v. ya bh* 0 After equal subtraction *ya gh* 4 *ya v* 0 *ca v. ya bh* 12
made, depressing both sides by the common divisor *ya,* and superadding

unity, the root of the first side of equation is *ya* 2 *ru* 1. Roots of the other side (ca v 12 *ru* 1) are investigated by the rule of the affected square, and are L2 G7 or *L*28 G97. "Least" root is a value of ca. Making an equation of a "greatest" root with *ya* 2 *ru* 1, the value of *ya* is obtained: viz. 3 or 48. Substitution being made with the respective values, the two quantities come out 1 and 5, or 20 and 76, and so forth.

CHAPTER IX
CONCLUSION

A particle of tuition conveys science to a comprehensive mind; and having reached it, expands of its own impulse. As oil poured upon water, as a secret entrusted to the vile, as alms bestowed upon the worthy, however little, so does science infused into a wise mind spread by intrinsic force.

It is apparent to men of clear understanding, that the rule of three terms constitutes arithmetic; and sagacity, algebra. Accordingly I have said in the chapter on Spherics:

224. "The rule of three terms is arithmetic; spotless understanding is algebra.[10] What is there unknown to the intelligent? Therefore, for the dull alone, it[11] is set forth."

225. To augment wisdom and strengthen confidence, read, do read, mathematician, this abridgment elegant in style, easily understood by youth, comprising the whole essence of computation, and containing the demonstration of its principles, replete with excellence and void of defect.

NOTES

1. *Rina* or *cshaya*, minus; literally debt or loss: negative quantity. *D'hana* or *swa*, plus; literally wealth or property: affirmative or positive quantity.

2. *Rasi*, quantity, is either *vyacta*, absolute, specifically known, (which is termed *rupa*, form, species;) or it is *avyacta*, indistinct, unapparent, unknown *(ajnyata)*. It may either be a multiple of the arithmetical unit, or a part of it, or the unit itself.

3. The sign only of the product is taught. All the operations upon the numbers are the same which were shown in simple arithmetic (*Līlāvatī* § 14–16).

4. Square and square-root.

5. As much as the divisor is diminished, so much is the quotient raised. If the divisor be reduced to the utmost, the quotient is to the utmost increased. But, if it can be specified, that the amount of the quotient is so much, it has not been raised to the utmost: for a quantity greater than that can be assigned. The quotient therefore is indefinitely great, and is rightly termed infinite.

6. Or else multiplying two.

7. *Ananta-rasi*, infinite quantity. *C'hahara*, fraction having cipher for its denominator.

8. *Yavat-tavat*, correlatives, quantum, tantum; quot, tot: as many, or as much, of the unknown, as this coefficient number. *Yavat* is relative of the unknown; and *tavat* of its coefficient.

The initial syllables of the *Sanscrit* terms enumerated in the text are employed as marks of unknown quantities; viz. *ya*, *ca*, *ni*, *pi*, *lo*, (also *ha*, *swe*, *chi*, &c. for green, white, variegated and so forth). Absolute number is denoted by *ru*, initial of *rupa* form, species. The letters of the alphabet are also used (ch. 6), as likewise the initial syllables of the terms for the particular things (§111).

9. *Mana*, *miti*, *unmana* or *unmiti*, measure or value.

10. *Vija*.

11. The solution of certain problems set forth in the section. The preceding stanza . . . premises, "I deliver for the instruction of youth a few answers of problems found by arithmetic, algebra, the pulverizer, the affected square, the sphere, and [astronomical] instruments."