Constructions!

Construct the following using Geometer's Sketchpad without using the "Construct Menu". Do them in any order. If you have completed one construction, it is acceptable to use it in another construction. Explain each of your steps in the process.

1 Point at midpoint of a given segment

2 Perpendicular line to a line through a given point on the line

3 Perpendicular line to a line through a given point not on the line

4 Parallel line to a line through a given point not on the line

5 Angle bisector of a given angle

6 Circle by center and radius

7 Circle through three points

Work on the following constructions. Using the "Construct Menu" is now acceptable (you have justified it with 1-7 above).

8 Draw a circle. Label the centre P. Locate a point on the circle. Label it A. Construct a tangent to ○P at A.

9 Draw a circle. Label the centre Q. Draw a point exterior to the circle. Label it B. Construct a tangent to ○Q containing B.

10 Construct a segment whose measure is twice the measure of a given segment.

11 Construct a segment whose measure is three times the measure of a given segment.

12 Construct a segment whose measure is the sum of the measures of two given segments.

13 Construct a segment whose measure is the difference of two given segments.

14 Draw two segments. Then, construct the geometric mean of the two segments. *or* Draw a rectangle. Then, construct a square with the same area as the given rectangle.

15 Given a segment of length defined to be one, construct a segment whose length is the square root of the length of a given segment.

16 Construct a square.

17 Construct two squares. Construct a square of equal area to the sum of the two squares.

18 Given a segment of length defined to be one, construct a segment whose length is the product of the measure of two given segments.

19 Given a segment of length defined to be one, construct a segment whose length is the quotient of the length of two given segments.

20 Prove that given a defined length of 1, we can construct a length equal to any rational number, and the square root of any length. Explain constructions not explained elsewhere.

21 Construct an angle whose measure is twice the measure of a given angle.

22 Construct an angle whose measure is half the measure of a given angle.

23 Construct an angle whose measure is the sum of the measures of two given angles.

24 Construct an angle whose measure is the difference of the measures of two given angles.

25 Prove that we can add, subtract, multiply by whole numbers, and divide by two given angles. Explain constructions not explained elsewhere in this assignment. Using this and other reasoning, what are the whole-number degree angle measures that are constructible?

26 Construct an equilateral triangle.

27 Construct a square.

28 Construct a regular hexagon.

29 Construct a regular octagon.

30 Using the fact that $\cos(72°) = \frac{\sqrt{5} - 1}{4}$, construct a regular pentagon. Using this construction, construct a regular 15-gon. Describe the set of the regular polygons *you* can construct. Explain heuristically how you would construct each of them. (Hint: The list should be infinite, so you do not need to give a step-by-step description for each. Explain the approach you would use in each case.)

31 Prove that $\cos(72°) = \frac{\sqrt{5} - 1}{4}$. Consider the isosceles triangle $\triangle ABC$ with $|AB| = |AC| = 1$ and $\angle BAC = 36°$. Find the length of $|BC|$. (Hint: Construct the angle bisector of $\angle ABC$.)

32 Construct an equilateral triangle. Construct a square of equal area.

33 Construct a regular hexagon. Construct a square of equal area.

34 Which regular polygons can you square (i.e. for which regular polygons can you construct a square of equal area)? Which polygons can you square?

35 Show that to square the circle, one must be able to construct a segment of length π. In the latter part of the nineteenth century, it was proven that π is transcendental.

36 Draw a square. Construct a square with twice the area.

37 Prove that if it is possible to double the cube, then it is possible to construct a segment of length $\sqrt[3]{2}$.