6. Many children have had fun with this one. The directions are to write your name on graph paper and see the patterns that occur with different lengths. You are finished when the last letter of your name appears on the last column, thus making a rectangle. The case for a 3-letter name is shown below for three different lengths.

4 columns					5 columns						7 columns							
P	A	T	P		P	A	T	P	A		P	A	T	P	A	T	P	
A	T	P	A		T	P	A	T	P		A	T	P	A	T	P	A	
T	P	A	T		A	T	P	A	T		T	P	A	T	P	A	T	

a. Using a sheet of "Other Base Graph Paper" at the end of this book, try your name with various lengths. Describe any patterns you see, your observations, and any questions.

Two questions that often come from the initial exploration are described below. Exploring them involves some wonderful opportunities to understand various aspects of number theory, as well as to develop your competence with the five process standards stated by NCTM. Read the two questions and then follow the directions immediately below the questions.

b. ***Question 1*** Background: You will have noticed that in some cases, there is a diagonal pattern going from bottom left to top right (as in the 4- and 7-column example) above, sometimes there is a diagonal pattern going the other way (as in the 5-column example), and sometimes there is no real diagonal pattern.

The question: Can you predict when a rectangle will have the first diagonal pattern, the second diagonal pattern, or no diagonal pattern, given the length of letters in the "name" and the length of the column?

c. ***Question 2*** Background: We can also look at the dimensions of the rectangle formed by one complete cycle. In the case of a 3-letter name, the rectangle for a 4-column pattern is 4×3, the rectangle for a 5-column pattern is 5×3, and the rectangle for a 7-column pattern is 7×3.

The question: Can you predict the dimensions of the rectangle, given the length of letters in the "name" and the length of the column? Note that there are many possibilities explaining the makeup of the finished rectangle—odd numbers, even numbers, prime numbers, numbers with common factors, and many more!

EXPLORATION 4.5 Cycles

We encounter cycles all the time—weather cycles, biological cycles, economic cycles, life cycles of plants and animals. The elementary and middle schools in Keene all follow a 6-day cycle. We see cycles in many words, such as *tricycle* and *bicycle*, and the words *circle* and *cycle* have a common root. Below are several problems that involve cycles. Solving them draws on important ideas from number theory.

1. Most carnivals and amusement parks have a Ferris wheel ride. Some larger ones have double Ferris wheels, and others have large and small Ferris wheel rides. Suppose two sisters decide to go on the big Ferris wheel, which makes one complete rotation in 50 seconds. A younger brother is afraid of the large wheel and goes on the small Ferris wheel with his father; the small Ferris wheel makes one complete rotation in 20 seconds.
   a. Assuming that both rides begin at the same time, when will the two sisters and the father and son be at the bottom again?
   b. What if the periods of the Ferris wheels were 45 seconds and 25 seconds?
   c. What if the periods were 80 seconds and 50 seconds?
   d. When will the two sisters and the father and son all be at the top at the same time with the 50- and 20-second Ferris wheels?
2. Planetary alignments
   a. In 1988 Earth, Jupiter, Saturn, and Uranus were all in alignment. When will they be in alignment again if the orbits of the three other planets are 12, 30, and 84 Earth years?
   b. Let's say that Earth, Mercury, and Venus are aligned at a certain time. How long will it be before all three are again aligned, if the yearly cycles of Mercury and Venus are 88 and 224 Earth days?
3. Our physical cycle completes one life span (or cycle) in 23 days. The emotional cycle lasts 28 days, and the intellectual cycle lasts 33 days. These cycles are hypothesized to rise and fall like sine waves in trigonometry. Let's say a person's physical, emotional, and intellectual cycle all peaked. When would be the next time all three would peak?
4. Your organization is having a fund raiser and will be selling hot dogs. You anticipate selling about 200 hot dogs. You go to the store and find that the hot dogs come in packages of 12, but the hot dog buns come in packages of 8. What number around 200 would give you the same number of hot dogs and hot dog buns?
5. An interactive museum has many programs that cycle throughout the day. Assume that all of the programs begin at 9 a.m.
   a. If you want to see two programs that have, respectively, 10- and 15-minute cycles, what is the earliest you could be finished?
   b. If you want to see three programs that have, respectively, 10-, 15-, and 20-minute cycles, what is the earliest you could be finished?
   c. If you want to see three programs that have, respectively, 10-, 12-, and 15-minute cycles, what is the earliest you could be finished?