Triangles," 1579). In both, mathematics is closely connected with astronomical and cosmological research. Interestingly, Viète accepted Ptolemaic rather than Copernican astronomy because he believed that the latter was not valid geometrically. His *Canon* was probably the first text in Western Europe to develop systematically plane and spherical triangle solutions using all six trigonometric lines, which later became functions. Viète based his trigonometric tables largely on the chords and half chords of Ptolemy's *Almagest* and al-Zarqālī's *Canones*. He divided the radius of his unit circle into 60 parts. He also presented decimal fractions independently of Stevin and helped make them standard in western Europe.

During the second period, Viète prepared most of the *In artem analyticem isagoge* ("Introduction to the Analytical Art," 1591). In this—his most important book—he brought together the ancient geometrical methods of Euclid, Archimedes, Apollonius, and Pappus with the numerical algebra of Diophantus, Cardano, Tartaglia, Bombelli, and Stifel. Viète's *Isagoge*, the first book on symbolic algebra in Western Europe, resembles a modern algebra text. It uses consonants to denote known quantities, vowels to denote unknowns, and has a syncopated form for powers, where A quadratus, A cubus, . . . represents A^2, A^3, ... For his pioneering work on symbolism Viète is often called the father of modern algebraic notation.

Viète made other contributions to algebra. In 1615, he contributed to the theory of equations in the treatise *De aequationum recognitione et emendatione* ("Concerning the Recognition and Emandation of Equations"). In 1646, Franz van Schooten published Viète's collected works, but they were already relegated to a secondary status in mathematics because of the appearance of Cartesian (analytic) geometry with its more felicitous symbols. All of Viète's books except this last one were printed and distributed at his expense.

57. From *In artem analyticem isagoge**

(The New Algebra)

FRANÇOIS VIÈTE

CHAPTER I. ON THE DEFINITION AND PARTITION OF ANALYSIS, AND ON THOSE THINGS WHICH ARE OF USE TO ZETETICS

In this chapter Viète refers to Pappus's distinction between analysis and synthesis, and between zetetic and poristic analysis, referring also to Euclid and Theon. There also should be, he writes, a third kind of analysis, the rhetic or exegetic,

so that there is a zetetic art by which is found the equation[2] or proportion between the magnitude that is being sought and the given things; a poristic art by which from the equation or proportion the truth of the required theorem is investigated, and an exegetic art by which from the constructed equation or proportion there is produced the magnitude itself that is being sought. And the whole threefold analytical art may be defined as the science of finding the truth in mathematics. But what truly belongs to the zetetic art is established by the art of logic through syllogisms and enthymemes,[3] of which the foundations are those very symbols[4] by which equations and proportions are obtained . . . The zetetic art, however, has its own form of proceeding, since it applies its logic not to numbers—which was the boring habit of the ancient analysis—but through a logistic which in a new way has to do with species.[5] This logistic is much more successful and powerful than the numerical one for comparing magnitudes with one another in equations, once the law of homogeneity has been established and there has been constructed, for that purpose, a traditional series or scale of magnitudes ascending or descending by their own nature from genus to genus, by which scale the degrees and genera of magnitudes in equations may be designated and distinguished.

CHAPTER II. ON THE SYMBOLS FOR EQUATIONS AND PROPORTIONS

Here Viète takes a number of postulates and propositions from Euclid, such as:

1. The whole is equal to the sum of its parts;

2. Things that are equal to the same thing are equal among themselves;

3. If equals are added to equals, the sums are equal; . . .

8. If like proportionals are added to like proportionals, then the sums are proportional;[6] . . .

15. If there are three or four magnitudes, and the product of the extreme terms is equal to either that of the middle one by itself or that of the middle terms, then these magnitudes are proportional.[7]

CHAPTER III. ON THE LAW OF HOMOGENEOUS QUANTITIES, AND THE DEGREES AND GENERA OF THE MAGNITUDES THAT ARE COMPARED

The first and supreme law of equations or of proportions, which is called the law of homogeneity, since it is concerned with homogeneous quantities, is as follows:

1. Homogeneous quantities must be compared to homogeneous quantities [*Homogenea homogeneis comparari*].

Indeed, it cannot be known how heterogeneous quantities can be affected among themselves, as Adrastus says.[8] Hence:

If a magnitude is added [*additur*] to a magnitude, it is homogeneous with it.

If a magnitude is subtracted [*subducitur*] from a magnitude, it is homogeneous with it.

If a magnitude is multiplied [*ducitur*] by a magnitude, the result is heterogeneous with both.

Since they did not, these ancient Analysts, attend to this, the result was much obscurity and blindness.

2. Magnitudes which by their own nature ascend or descend proportionally from genus to genus are called scalars.[9]

The first of the scalar magnitudes is side or root [*latus seu radix*].[10]

The second is square [*quadratum*].

The third is cube.

The fourth is squared square [*quadrati-quadratum*].

The fifth is squared-cube . . .

The ninth is cubed-cubed-cube.

And the further ones can from here be named by this series and method . . .

The genera of the magnitudes that we have to compare so that they may be named in the order of the scalars are:

(1) Length and breadth [*longitudo, latitudo*],
(2) Plane,
(3) Solid,
(4) Plane-plane,
(5) Plane-solid . . .
(9) Solid-solid-solid.

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And the further ones can be named from here by this series and method.

[*text omitted*]

5. In a series of scalars, the degree in which the magnitude stands compared to the side is called the power [*potestas*]. The other inferior scalars are called parodic[11] grades to this power.

6. The power is pure when it is free from affection. By affection is meant that a homogeneous magnitude is mixed with a magnitude of lower power together with a coefficient.[12]

To this Van Schooten adds: "A pure power is a square, cube . . . But an affected power is in the second grade: a square together with a plane composed of a side and a length or breadth;[13] in the third grade: a cube together with a solid composed of a square and a length or latitude."

7. Adjunct magnitudes which multiply scalars lower in relation to a certain power and thus produce homogeneous magnitudes are call subgradual.

Van Schooten adds: "Subgraduals are length, breadth, plane, solid, plane-plane, etc. Thus if there be a squared square with which is mixed a plane-plane which is the side multiplied with a solid, then the solid will be a subgradual magnitude, and in relation to the squared square the side will be a lower scalar."

CHAPTER IV. ON THE RULES FOR THE CALCULATION BY SPECIES [*logistica speciosa*]

Numerical calculation [*logistica numerosa*] proceeds by means of numbers, reckoning by species by means of species or forms of things, as, for instance, the letters of the alphabet.

Van Schooten adds: "Diophantus operates with numerical calculation in the thirteen books of his *Arithmetica*, of which only the first six are extant, and are now available in Greek and Latin, illustrated by the commentaries of the very erudite Claude Bachet.[14] But the calculation by species has been explained by Viète in the five books of his *Zetetics*,[15] which he has chiefly arranged from selected questions of Diophantus, some of which he solves by his own peculiar method. Wherefore, if you wish to understand with profit the distinction between the two logistics, you must consult Diophantus and Viète together." He then compares specifically certain problems of Diophantus with his and with Viète's solutions.

There are four canonical rules for the calculation by species.

RULE I

To add a magnitude to a magnitude. Take two magnitudes A and B. We wish to add the one to the other. But, since homogeneous magnitudes cannot be affected to heterogeneous ones, those which we wish to add must be homogeneous magnitudes. That one is greater than the other does not constitute diversity of genus. Therefore, they may be fittingly added by means of a coupling or addition; and the aggregate will be A plus B, if they are simple lengths or breadths. But if they stand higher in the scale, or if they share in genus with those that stand higher, they will be denoted in the appropriate way, say A square plus B plane, or A cube plus B solid, and similarly in further cases.

However, the Analysts are accustomed to indicate the affection of summation by the symbol +.

RULE II

To subtract a magnitude from a magnitude. **This leads in an analogous way to $A - B$, A square $-$ B square, A is larger than B, also to rules such as $A - (B + D) = A - B - D$; Viète writes = instead of our $-$.**

RULE III

To multiply a magnitude by a magnitude. Take two magnitudes A and B. We wish to multiply the one by the other.

Since then a magnitude has to be multiplied by a magnitude they will by their multiplication produce a magnitude heterogeneous with respect to each of them; their product will rightly be designated by the word "in" or "under" [*sub*], e.g., A in B, which will mean that the one has been multiplied by the other, or, as others say, under A and B, and this simply when A and B are simple lengths or breadths.[16]

But if the magnitudes stand higher in the scale, or if they share in genus with these magnitudes, then it is convenient to add the names themselves, e.g., A square in B, or A square in B plane solid, and similarly in other cases.

If, however, among magnitudes that have to be multiplied, two or more are of different names, then nothing happens in the operation. Since the whole is equal to its parts, the products under the segments of some magnitude are equal to the product under the whole. And when the positive name [*nomen adfirmatum*] of a magnitude is multiplied by a magnitude also of positive name, the product will be positive, and negative [*negatum*] when it is negative.[17]

From which precept it follows that by the multiplication of negative names the product is positive, as when $A - B$ is multiplied by $D - G$; since the product of the positive A and the negative G is negative, which means that too much is taken away [and similarly negative B into positive]. Therefore, in compensation, when the negative B is multiplied by the negative G the product is positive.[18]

The denominations of the factors that ascend proportionally from genus to genus in magnitude behave, therefore, in the following way:

A side multiplied by a side produces a square,
A side multiplied by a square produces a cube . . .
And conversely, a square multiplied by a side produces a cube . . .
A solid multiplied by a solid-solid produces a solid-solid-solid,
And conversely, and so on in that order.

RULE IV

To divide a magnitude by a magnitude. This leads in an analogous way to such expressions as $\frac{B \text{ plane}}{A}$, $\frac{B \text{ cube}}{A \text{ plane}}$ and so forth. Furthermore to add $\frac{Z \text{ plane}}{G}$ to $\frac{A \text{ plane}}{B}$; the sum will be

$$\frac{G \text{ in } A \text{ plane} + B \text{ in } Z \text{ plane}}{B \text{ in } G}.$$

To multiply $\frac{A \text{ plane}}{B}$ by Z; the result will be $\frac{A \text{ plane in } Z}{B}$.

CHAPTER V. CONCERNING THE LAWS OF ZETETICS

The way to do Zetetics is, in general, directed by the following laws:

1. If we ask for a length, but the equation or proportion is hidden under the cover of the data of the problem, let the unknown to be found be a side.
2. If we ask for a plane . . . let the unknown to be found be a square.

[*text omitted*]

9. If the element that is homogeneous under a given measure happens to be combined with the element that is homogeneous in conjunction, there will be antithesis.

These laws amount to introducing (1) x, (2) x^2, (3) x^3, (4) the law of homogeneity, as in $x = ab$; and to (5) denoting the unknown by vowels $A, E, \ldots$ and the given magnitudes by consonants, $B, G, D, \ldots$, (6) constructing $x^2 = ab + cd$, or, as Viète writes it: A square equal to B in C + D in F; (7) forming $ax \pm bx$ ("homogeneous in conjunction"); (8) forming $x^3 + ax^2 - bx^2 = c^2d + e^3$; (9) passing from $x^3 + ax^2 + bx^2 - c^2d + e^2f = g^3$ to $x^3 + ax^2 - bx^2 = c^2d - e^2f + g^3$ ("antithesis"). Then Viète continues with Propositions marked (10) to (12), which state that an equation is not changed by antithesis, by hypobibasm, and by parabolism. Hypobibasm means dividing

by the unknown, as passing from $x^3 + ax^2 = b^2x$ to $x^2 + ax = b^2$, parabolism is dividing by a known magnitude. Nos. (13) and (14) deal with the relation of equations to proportions.

These are the titles of the next chapters:

VI. Concerning the examination of theorems by means of the poristic art.

VII. Concerning the function of the rhetic art.

VIII. The notation of equations and the epilogue to the art.[19]

This chapter ends as follows:

29. Finally, the analytic art, now having been cast into the threefold form of zetetic, poristic, and exegetic, appropriates to itself by right the proud problem of problems, which is

THERE IS NO PROBLEM THAT CANNOT BE SOLVED.[20]

NOTES

1. Footnote omitted.

2. Viète writes *aequalitas*, equality, but the term "equation," now used, seems to fit the meaning better. The stress on proportion is due to the respect in which Book V of Euclid's *Elements* was held as a model whereby the contradiction between arithmetic and geometry could be overcome by rigorous mathematical reasoning.

3. An enthymeme is a syllogism incompletely stated, perhaps by leaving out the major or the minor premise; for example, in "John is a liar, therefore he is a coward," the premise, "every liar is a coward," is omitted.

4. Symbols, *symbola*, had here more the meaning of typical rules or stipulations.

5. Hence the name "logistica speciosa" for Viète's new type of calculation. The term *logistike* was used by the Greeks for the art of calculation, in contrast to *arithmetike*, number theory. Viète's term "species" is probably the translation of Diophantus' *eidos*, the term in a particular expression, primarily in reference to the specific power of the unknown it contains. See further the J. Winfree Smith translation of the *Isagoge*, pp. 21-22.

6. If $a : b = c : d$, then $(a + c) : (b + d) = a : b = c : d$.

7. If a, b, c, d are such that either $ac = b^2$ or $ad = bc$, then either $a : b = b : c$ or $a : b = c : d$.

8. Reference to a reference in Theon: "For Adrastus says that it is impossible to know how heterogeneous magnitudes may be in a ratio to one another." Who Adrastus was does not seem to be known.

9. *Scalares* means "ladder magnitudes," literally, steps or rungs of a ladder. Viète follows Diophantus in the naming of the powers. The term *scalar*, of vector-analysis fame, is due to W. R. Hamilton (1853).

10. This is the *cosa*, or *res*, of the cossists, hence x in our notation. The next scalars are x^2 (square), x^3, x^4, and so forth. In Viète these quantities have dimensions.

11. *Parodic* is from Greek *para, hodas*, on the way, coming up.

12. x^5 is pure, $x^5 + ax^4$ is affected.

13. x^2 is pure, $x^2 + ax$ is affected.

14. Bachet's edition of Diophantus is of 1621, and was the inspiration of Fermat's work on numbers (see selection I.6).

15. In this work of 1593 Viète gives many examples of his *logistica speciosa*.

16. In arithmetic the custom was to use "in": *ducta in*; in geometry, "under": a rectangle is "under" its sides.

17. + in + is +; + in − is −.

18. $(A - B)(D - G) = AD - AG - BD + BG$.

19. Chapter VI mentions the retracing of the zetetic process by synthesis; Chapter VII the special application of the analytic art, after solution, to special arithmetic and geometric problems. Here Viète speaks of the "exegetic art." Chapter VIII is the discussion of different possible expressions and equations, stressing homogeneity. There are 29 rules.

20. Quod est, Nullum non problema solvere.