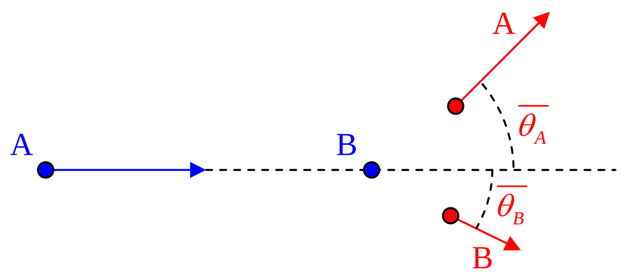
(This example is a little artificial, because it does not conserve energy.)

Proton A moving at 0.9*c* collides with a stationary proton B in an elastic collision. Proton A is deflected by an angle of 45°, leaving with speed 0.7*c*. At what angle and speed did proton B leave?

Notation:

- Use plain variables for before collision, and "overbar" variables for after collision.
- Use *v* to denote speed (always positive), and put signs for directions explicitly into equations.



(1) Momentum conservation gives $x: \quad \gamma_A m v_A = \overline{\gamma}_A m \overline{v}_{Ax} + \overline{\gamma}_B m \overline{v}_{Bx}$

$$= \overline{\gamma}_A m \overline{v}_A \cos 45° + \overline{\gamma}_B m \overline{v}_B \cos \overline{\theta}_B$$

$$y: \quad 0 = \overline{\gamma}_A m \overline{v}_{Ay} - \overline{\gamma}_B m \overline{v}_{By}$$

$$= \overline{\gamma}_A m \overline{v}_A \sin 45° - \overline{\gamma}_B m \overline{v}_B \sin \overline{\theta}_B$$

(2) Rearrange each equation to have A variables on one side and B variables on the other. Then divide the two equations to get $\tan \overline{\theta}_B = \dfrac{\overline{\gamma}_A m \overline{v}_A / \sqrt{2}}{\gamma_A m v_A - \overline{\gamma}_A m \overline{v}_A / \sqrt{2}} = \dfrac{\overline{\gamma}_A \overline{v}_A}{\sqrt{2}\gamma_A v_A - \overline{\gamma}_A \overline{v}_A}$

($\overline{\gamma}_B m \overline{v}_B$ cancels out.)

(3) $\gamma_A v_A = \dfrac{0.9c}{\sqrt{1-0.9^2}} = 2.0647c \quad , \quad \overline{\gamma}_A \overline{v}_A = \dfrac{0.7c}{\sqrt{1-0.7^2}} = 0.9802c$

(4) $\tan \overline{\theta}_B = \dfrac{0.9802}{2.0647\sqrt{2} - 0.9802} = 0.5053 \quad , \quad \overline{\theta}_B = 26.81°$

(5) from the *y* conservation equation, $\overline{\gamma}_B \overline{v}_B = \overline{\gamma}_A \overline{v}_A \dfrac{\sin 45°}{\sin 26.81°} = 1.5368c$

(6) $\overline{v}_B = \dfrac{1.5368c}{\sqrt{1+1.5368^2}} = 0.8382c$