Selected Topics

THE STANDARD MODEL

Standard Model

- Reconciles quantum mechanics with relativity
  - Each particle has a corresponding antiparticle

Types of Particles

- Fermions
  - Spin of ½ integer
  - Follow Fermi-Dirac statistics
  - Pauli Exclusion Principle
- Bosons
  - Integer spin
  - Follow Bose-Einstein statistics
  - Responsible for 4 fundamental forces

Fermions: Leptons

- Don't interact via strong force

Name	Symbol	Antiparticle	Charge (e)	Mass (MeV/c^2)
Electron	e^-	e^+	−1	0.511
Electron neutrino	ν_e	$\bar{\nu}_e$	0	< 0.000 0022
Muon	μ^-	μ^+	−1	105.7
Muon neutrino	ν_μ	$\bar{\nu}_\mu$	0	< 0.170
Tau	τ^-	τ^+	−1	1,777
Tau neutrino	ν_τ	$\bar{\nu}_\tau$	0	< 15.5

Fermions: Quarks

- Interact via strong force
- Cannot exist in unbound states
- Have 6 "flavors"

Name	Symbol	Antiparticle	Charge (e)	Mass (MeV/c^2)
up	u	$\bar{u}$	$+\frac{2}{3}$	1.5–3.3
down	d	$\bar{d}$	$-\frac{1}{3}$	3.5–6.0
charm	c	$\bar{c}$	$+\frac{2}{3}$	1,160–1,340
strange	s	$\bar{s}$	$-\frac{1}{3}$	70–130
top	t	$\bar{t}$	$+\frac{2}{3}$	169,100–173,300
bottom	b	$\bar{b}$	$-\frac{1}{3}$	4,130–4,370

Bosons

Name	Symbol	Antiparticle	Charge (e)	Spin	Mass (GeV/c^2)	Interaction mediated
Photon	γ	Self	0	1	0	Electromagnetism
W boson	W^-	W^+	−1	1	80.4	Weak interaction
Z boson	Z	Self	0	1	91.2	Weak interaction
Gluon	g	Self	0	1	0	Strong interaction
Higgs boson	H^0	Self	0	0	125.3	Mass
Graviton	G	Self	0	2	0	Gravitation

Ways to describe a particle

- Color
  - Strong nuclear force reactions
  - Red, green, blue
- Flavor
  - Quarks, up- or down-type
- Generation
  - Quarks
    - 1) u,d
    - 2) c, s
    - 3) t, b

Decay: Which force is responsible?

- Strong Force
  - Lifetimes around $10^{-23} s$
- Electromagnetic Force
  - Lifetime of $10^{-18} s$ to $10^{-16} s$
  - Generally a photon is emitted
- Weak Force
  - Lifetime of $10^{-10} s$ to $10^{-8} s$
  - Generally a neutrino is emitted
  - Beta decay

RECENT DISCOVERIES

Higgs Boson

- Responsible for giving mass to all matter
  - Spontaneous symmetry breaking
- Theorizes supersymmetry
  - Every particle has a superpartner with the same charge, but a difference in spin by 1/2

Neutrinos

- Have mass!
- Observed by studying neutrino oscillations
  - Emitted neutrinos detected as different “flavor”
- Less than 1/1000th mass of the electron

NUCLEAR PHYSICS: BOUND STATES

Composite Particles

Composite particles are made up of elementary particles

- Composites: protons, neutrons, pions
- Composites of composites: atomic nuclei, atoms, molecules

In quantum field theory, protons and neutrons are "teeming seas of quarks and gluons constantly popping in and out of existence"

Gluons – bosons that carry force that binds quarks together

Quark Model and Bound States

At low energies – where nuclear physics applicable:

- Proton – bound state of two up quarks and a down quark (uud)
  - Total charge of 2(2/3)-1/3=+1
- Neutron – bound state of two down quarks and an up quark (udd)
  - Total charge of 2(-1/3)+2/3=0

Due to confinement, there are no free quarks in nature. They instead collect into bound states, like protons and neutrons.

All of these bound states are color-neutral (color singlets).

Heavier bound states

Can be formed by colliding particles at higher and higher energies

Some can be considered excited states of nucleons.

ex: first excited state of proton has same quark content but can be considered a distinct particle as much heavier

Mesons and Baryons

Bound states of quarks generally fall into two categories:

Mesons	Baryons
Composite bosons	Composite fermions
Made of quark and antiquark	Made of three quarks
Either spin-1 or spin-0	Either spin-3/2 or spin-1/2
Pions, kaons, eta-mesons, rhos, phis, K*'s	Protons, neutrons, lambdas

Color introduced to explain quark content of some baryons

- Baryon of three identical quarks violates Pauli exclusion principle as supposed to be a fermion but has symmetric wavefunction – fixed by putting quarks in antisymmetric color state (color singlet)

Sample Problem

Without the hypothesis of quark color, the quark model would be unable to explain the existence of which of the following spin-3/2 baryons? You may assume the quarks have zero relative orbital angular momentum. (#91 from first practice exam in textbook)

a) udd

b) uud

c) uuu

d) uds

e) all of the above baryons are allowed

Decay

$^{A}_{Z}X$ X = atomic symbol

Z = atomic number (number of protons)

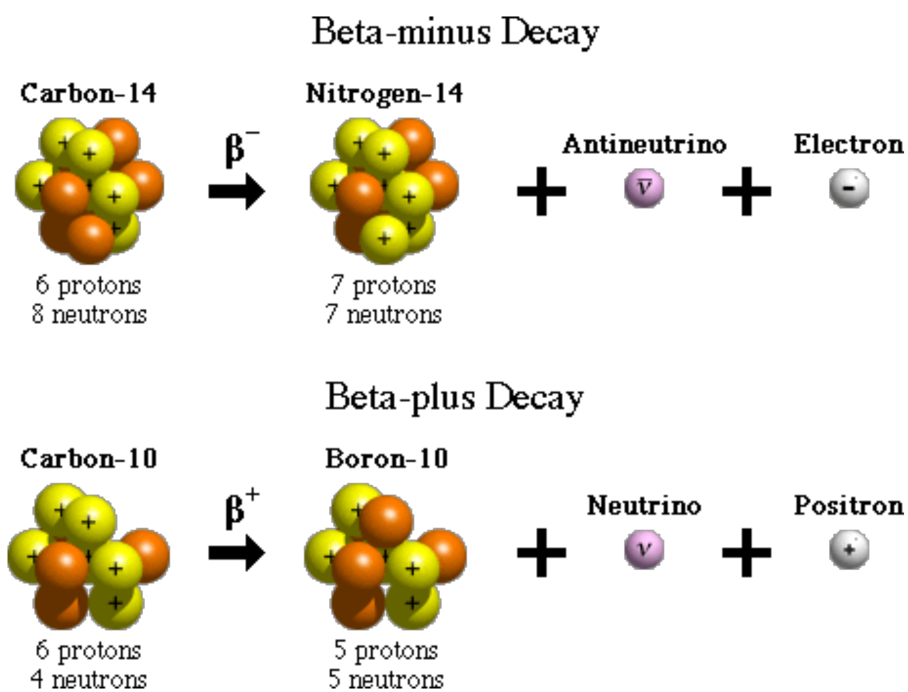
A = atomic mass number (number of protons and neutrons)

Neutron decay: $n \rightarrow p + e^{-} + \bar{\nu}_e$

Free neutrons decay (lifetime of about 15 min), but in nuclei, interactions with protons prevent this from happening immediately.

Useful to remember: typical nuclear diameters are femtometers

Beta decay



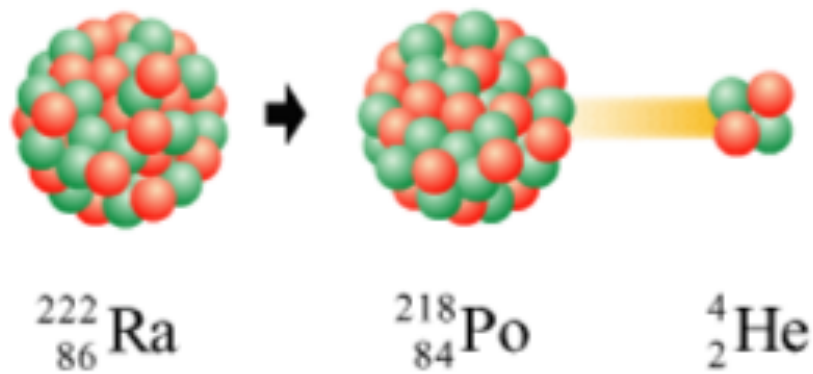
$$ {}^{A}_{Z}x \longrightarrow {}^{A}_{Z+1}y + {}^{0}_{-1}e + {}^{0}_{0}\overline{\nu_e} $$

$$ {}^{A}_{Z}x \longrightarrow {}^{A}_{Z-1}y + {}^{0}_{+1}e + {}^{0}_{0}\nu_e $$

Alpha decay

As nuclei get bigger, electromagnetic force starts to cancel strong force

$$^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$$



Gamma decay

The emission of photons from an excited state of a nucleus

- Does not change the composition of the nucleus

$$^{240}_{94}Pu \rightarrow ^{240}_{94}Pu + ^{0}_{0}\gamma$$

Fission and Fusion

- Fission – process in which large nuclei break apart
- Fusion – process in which small nuclei join together
  - Requires extreme temperatures and pressures
  - Can release large amounts of energy

Sun and hydrogen bomb are powered by fusion reactions

Genesis of heavy elements in early universe: light nuclei fused as result of supernovas up to iron (most stable nucleus). Heavier nuclei become more and more unstable – past lead all nuclei will eventually decay.

Fusion in the Sun

$$\begin{pmatrix} {}^1H + {}^1H \rightarrow {}^2H + e^+ + \upsilon_e \\ e^+ + e^- \rightarrow \gamma + \gamma \end{pmatrix} \times 2$$

$$\Downarrow$$

$$\left({}^2H + {}^1H \rightarrow {}^3He + \gamma\right) \times 2$$

$$\Downarrow$$

$${}^3He + {}^3He \rightarrow {}^4He + {}^1H + {}^1H$$

SYMMETRY AND CONSERVATION LAWS

Conservation laws

Everything that can happen will, unless forbidden by symmetry or conservation law.

An electron, as the lightest free negatively charged particle, can't decay due to conservation of charge. As the proton is heavier than the positron, why doesn't it decay?

- Conservation of baryon number:
  - Baryons get baryon number +1, anti-baryons get -1, all else get 0

Why is an extra neutrino always produced in beta decay?

- Conservation of lepton number:
  - Three separate conservations laws: electron number, muon number, tau number
  - Lepton and its associated neutrino given lepton number +1, corresponding antiparticles get -1, other stuff gets 0

Using these laws: decay modes of mu μ^-

Conservation of mu number:

- Muon has mu number of +1, so need lighter particle with mu number of +1 among decay products (mu neutrino)

$$\mu^- \rightarrow \qquad \nu_\mu$$

Using these laws: decay modes of mu μ^-

Conservation of charge:

- Muon has charge of -1, so need lighter particle with charge -1 among the decay products (electron)

$$\mu^- \rightarrow e^- + \nu_\mu$$

Using these laws: decay modes of mu μ^-

Conservation of electron number:

- Muon has electron number of zero and already have electron with electron number +1, so need a neutral particle with electron number of -1 (anti-electron neutrino)

$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$$

This is one decay mode. All other decay modes must contain extra pairs of particles with net charge and lepton number of 0 (dominant decay mode has smallest number of decay products)

Sample Problem

Which of the following is a possible decay mode of the π^+? (Note: $\overline{\nu}$ denotes an antineutrino.)

(A) $e^+\overline{\nu}_e$

(B) $e^+\nu_e$

(C) $\mu^+\overline{\nu}_\mu$

(D) $\mu^-\nu_e$

(E) $\mu^-\overline{\nu}_\mu$

Discrete Symmetry Operations

- P (parity): reverses orientation of space; takes config. to mirror image
- C (charge conjugation): exchanges particles and antiparticles
- T (time-reversal)

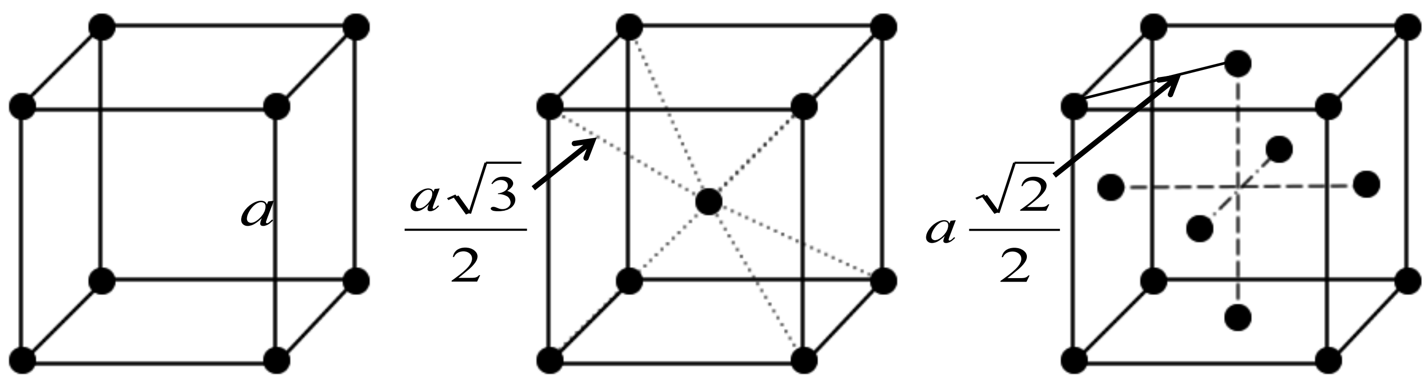
All Lorentz-invariant local quantum field theories must be symmetric under the combined action of these operations (CPT theorem)

However, Standard Model doesn't respect each of these individually

CRYSTAL STRUCTURE

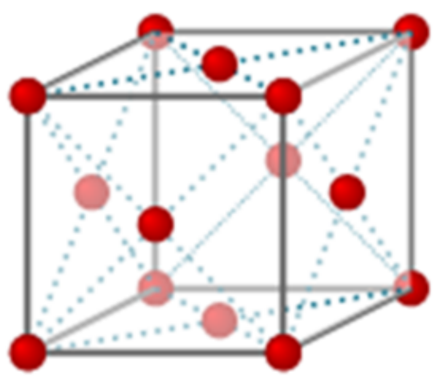
Crystal Lattice: Unit Cells

- Repetition – infinite structure
- Tessellate in all directions to build crystals
- Conventional unit cells (main three):

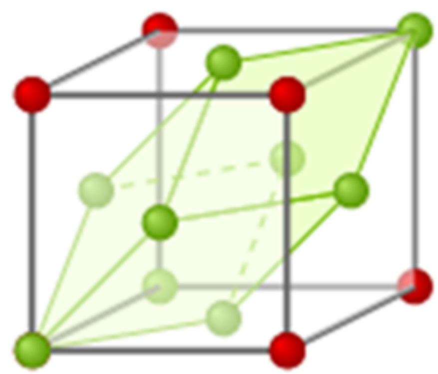


Crystal Lattice: Primitive Cells

- Conventional unit cell isn't always the smallest repeating pattern
- Smallest = least number of atoms



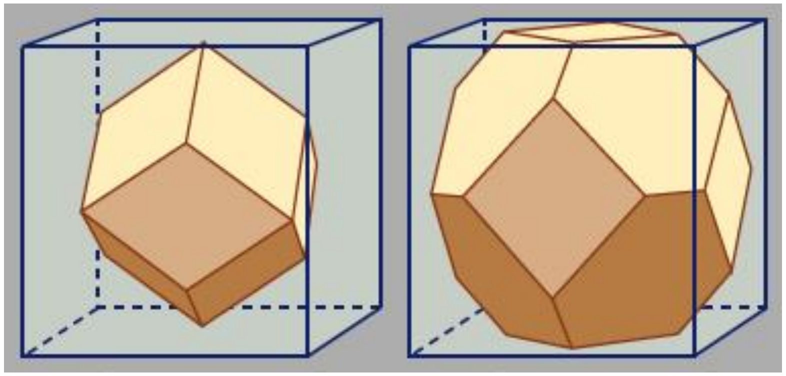
Conventional unit cell
(FCC)



Primitive unit cell
(FCC)

GRE Material

- Given the primitive cell volume:
- Find the volume of the conventional unit cell
- Find the inter-atomic distance



$$V_{Pf} = \frac{1}{4} V_{Uf}$$

$$V_{Pb} = \frac{1}{2} V_{Ub}$$

$$d_{fcc} = a\sqrt{2}/2$$

$$d_{bcc} = a\sqrt{3}/2$$

Reciprocal Lattice Structures

- Fourier transform of the original lattice
- Also called the dual lattice

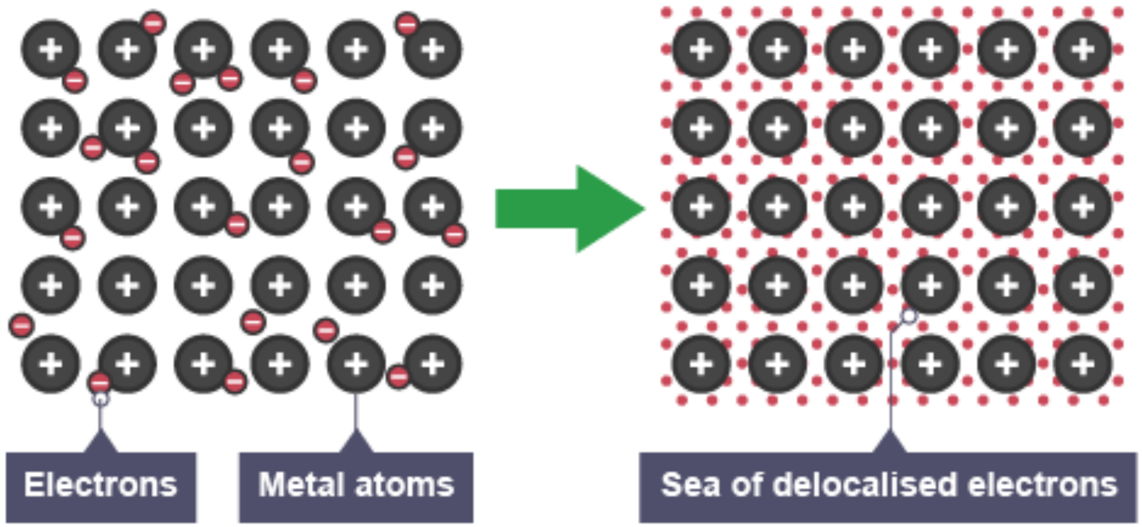
- Primitive unit cell for reciprocal lattice
  - Called the (first) Brillouin zone

- BCC/FCC lattices swap primitive unit cells

ELECTRON THEORY

Electron Theory of Metals

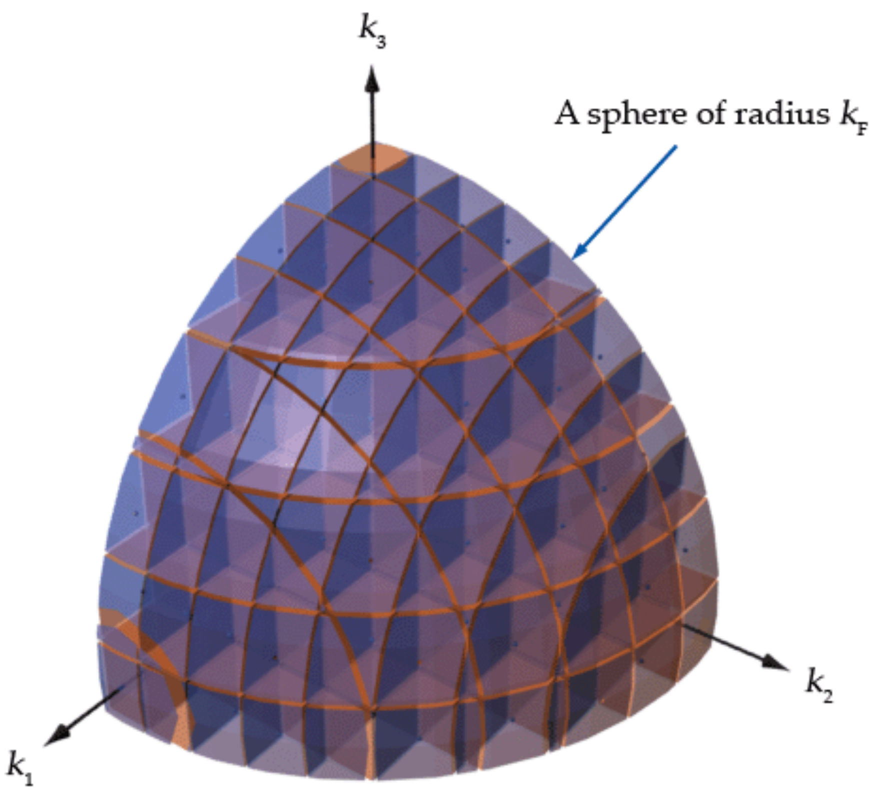
- Electrons in a crystal structure (metallic solids) are delocalized



Fermi Energy

- Classical mechanics: at 0 K, no motion, zero energy
- BUT, electrons are quantum particles (fermions)
- Pauli exclusion principle – can have two electrons with zero momentum, all others are moving

Fermi Energy



At 0 K, Electrons fill up a sphere in p-space

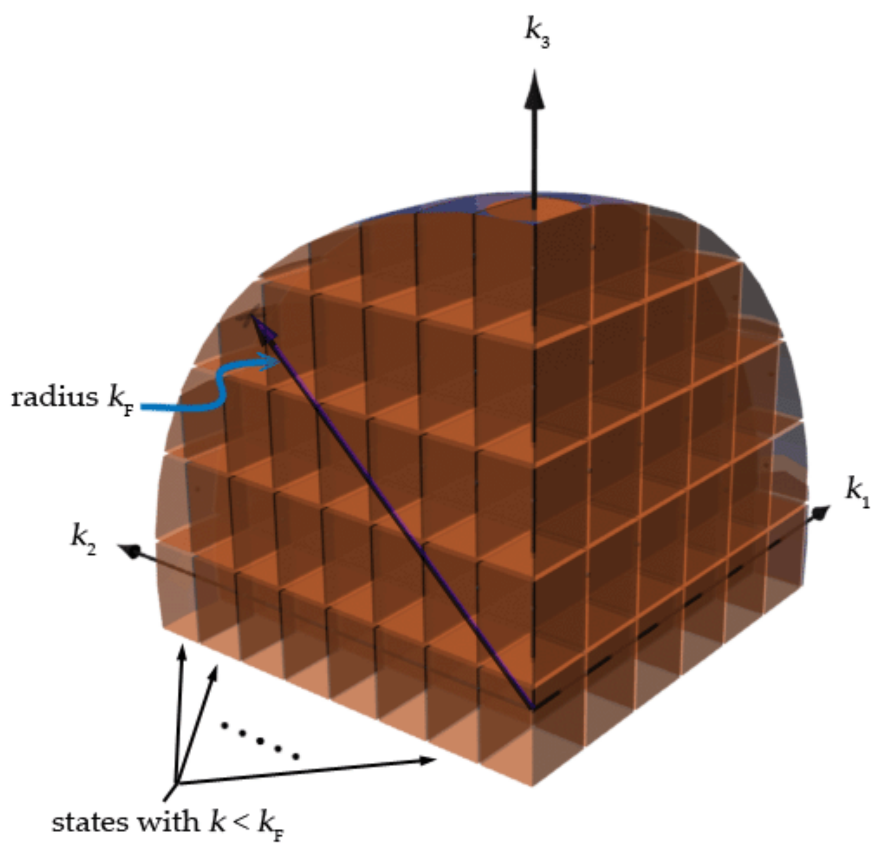
$$|\vec{k}| = k_F$$

Fermi wave vector

$$p_F = \hbar k_F$$

$$E_F = \hbar^2 k_F{}^2 / 2m$$

Fermi Energy



- k_f can be put in terms of the number density of electrons n

$$k_F = (3\pi^2 n)^{1/3}$$

- Using E_F from before

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$$

- Know the power laws

Other Derivations

- Density of states: number of possible free electron states at given energy E

$$\rho(E) = \frac{V\sqrt{2}}{\pi^2 \hbar^3} m^{3/2} \sqrt{E} \qquad \rho(E) \propto m^{3/2} \sqrt{E}$$

- Integrate for total number of electrons N

$$N = \int_0^{E_F} \rho(E) dE$$

CONDUCTORS

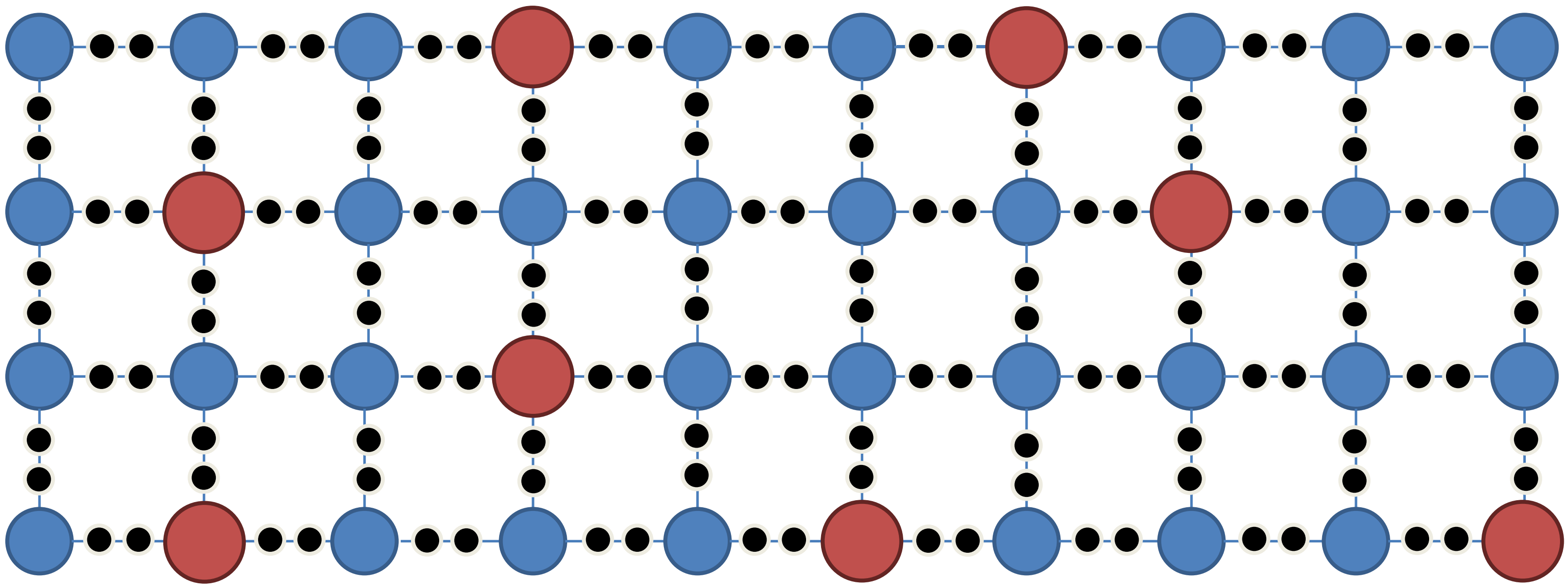
Semiconductors

Definition: Materials that are not as conductive as metals and have resistivity which decreases as temperature increases.

Common example: Silicon

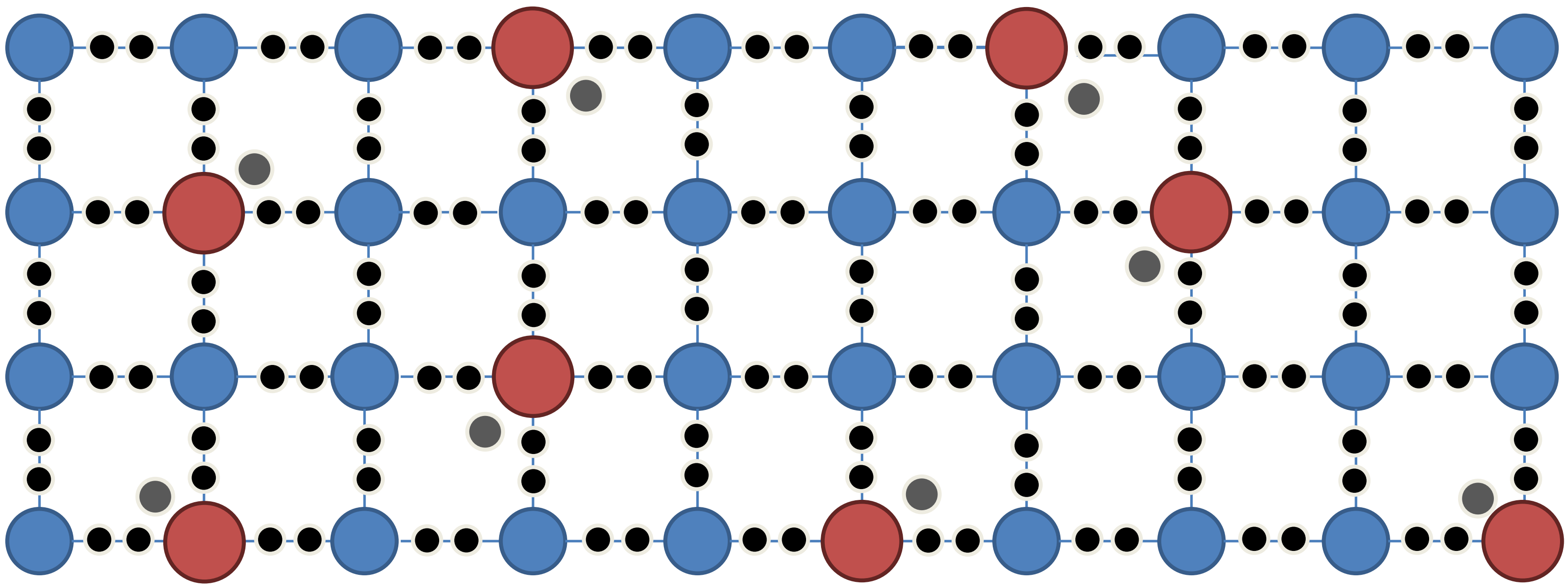
Semiconductors

The conductivity of a semiconductor is due to impurities within a lattice structure. Creating this impurities is known as “doping.”



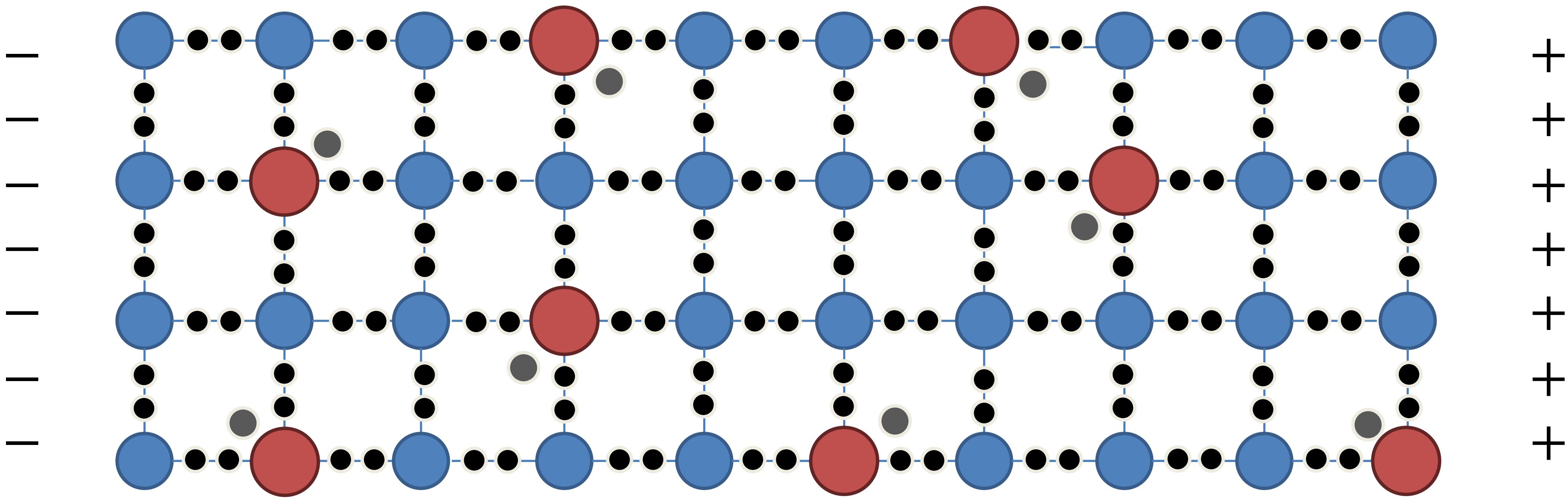
Semiconductors

One type of doping is called n-doping, where some Silicon atoms are replaced with atoms that have more electrons in their valence shell, such as Phosphorus.



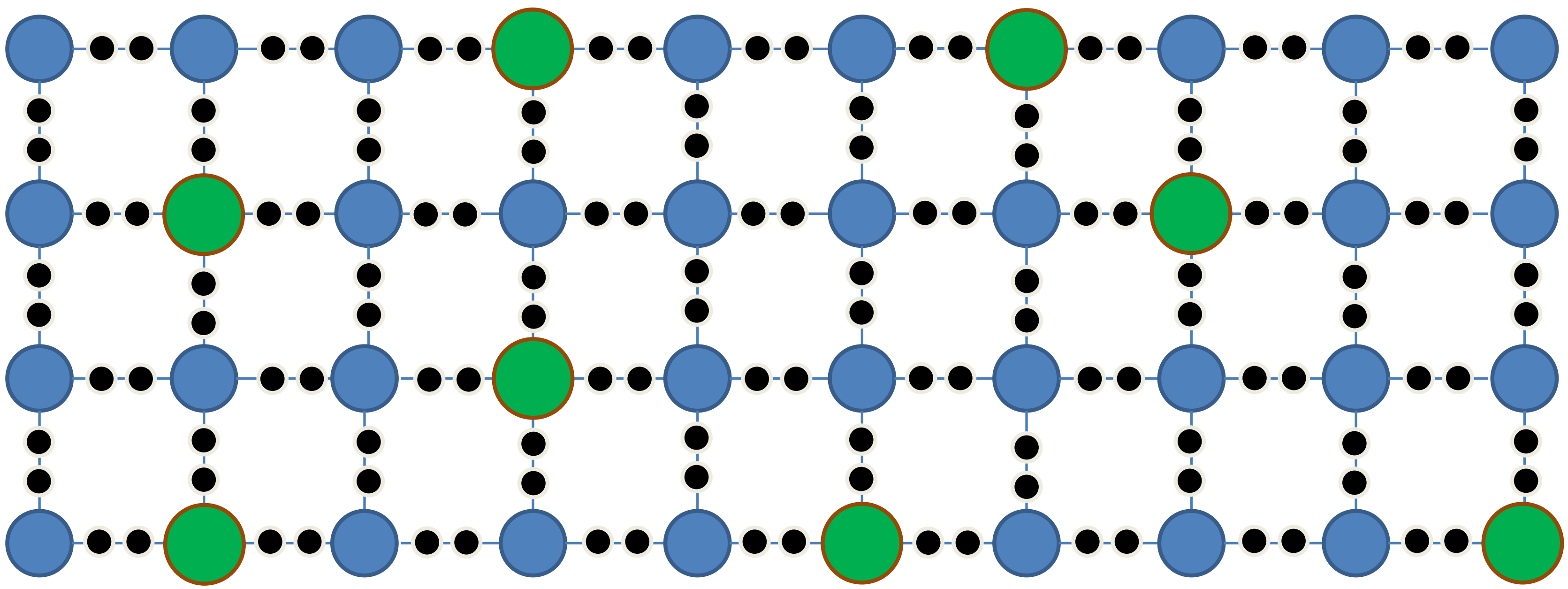
Semiconductors

Now, apply an electric potential across this lattice and see how the electrons in the conduction band are free to move away from the negative end and toward the positive end.



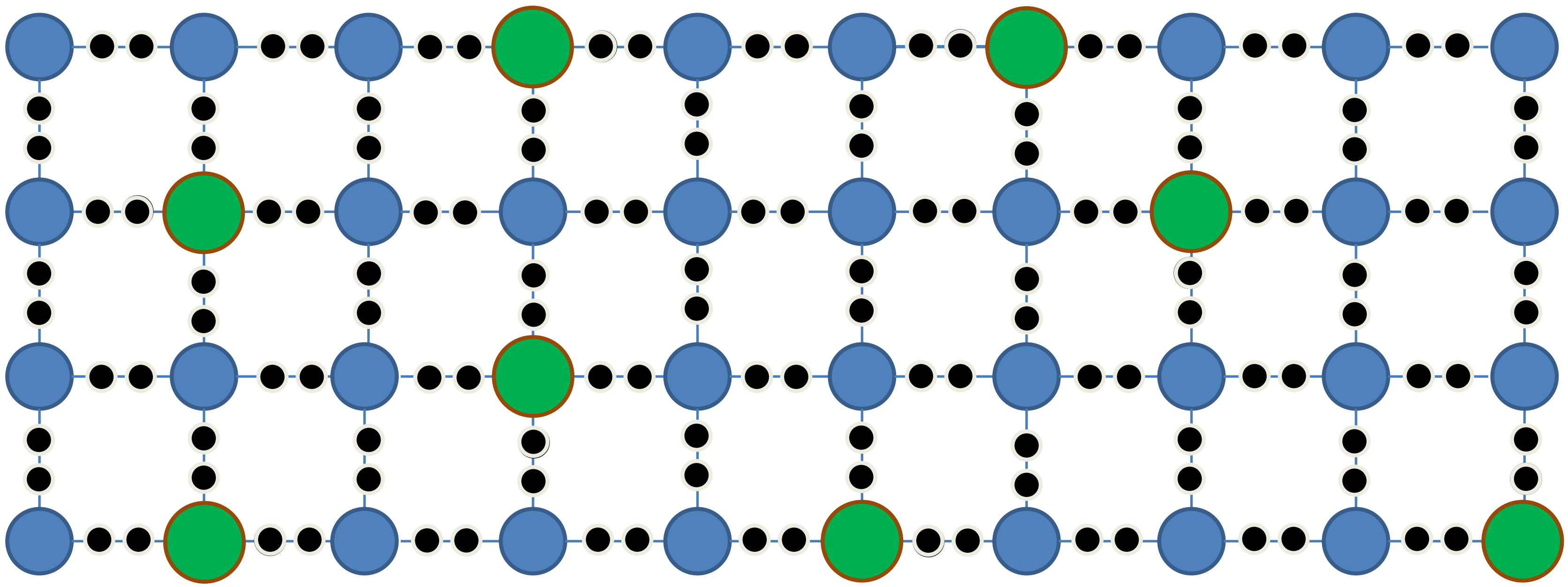
Semiconductors

Another type of doping is called p-doping, where some Silicon atoms are replaced with atoms that have less electrons in their valence shell, such as Boron.



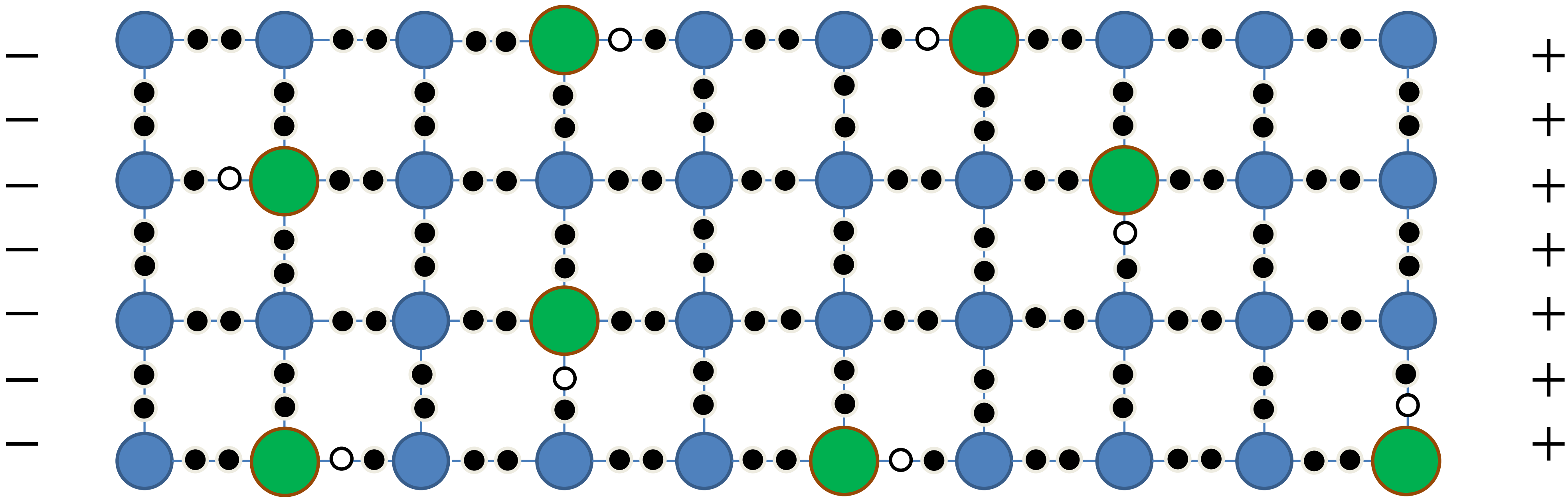
Semiconductors

Since there are less electrons in the valence shell, the lattice now contains many "electron holes," which can be thought of as positively-charged particles, even though they are not.



Semiconductors

With a potential applied across the lattice, these electron holes “move” toward the negative end. This movement is due to nearby electrons filling the holes as they move toward the positive end.

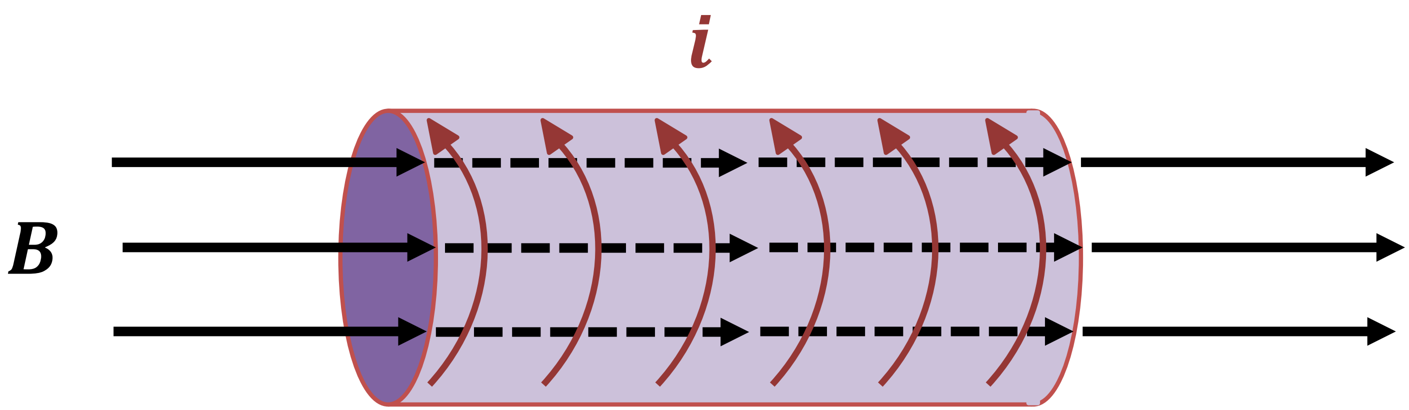


Superconductors

These are phenomena of quantum mechanics in which a material is cooled to extreme temperatures, and its electrical resistance becomes zero, giving it infinite conductivity.

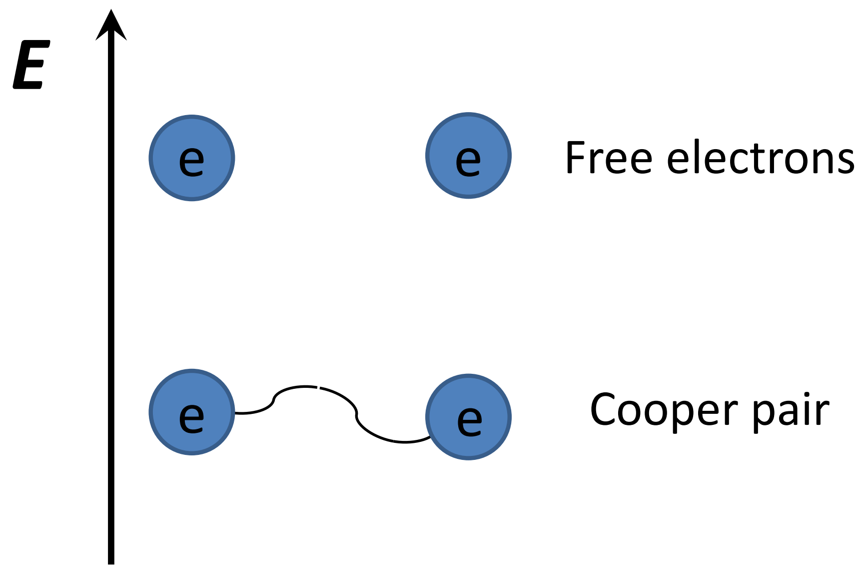
Superconductors

One result of cooling a material into a superconductor is the expulsion of magnetic fields within the conductor- *even if the field is present before the cooling begins*. This is known as the Meissner Effect.



Superconductors

Superconductivity can be explained by Cooper Pairs. A Cooper Pair is the result of two electrons bound together over several nanometers. They are known to be bound because the energy of the pair is lower than the Fermi energy (Fermi energy = energy of a single fermion at absolute zero).



Superconductors

It is important to keep in mind that although a Cooper Pair is composed of two fermions, it is in fact a boson. What this means is that these pairs have full-integer spins and can therefore occupy the same state. In other words, they can superimpose on one another.

ASTROPHYSICS

The Scale Factor a(T)

A(T) = Scale factor
= wavelength of photon
= wavelength at T

$$\frac{\lambda_o}{\lambda_T} = \frac{A(Today)}{A(T)}$$

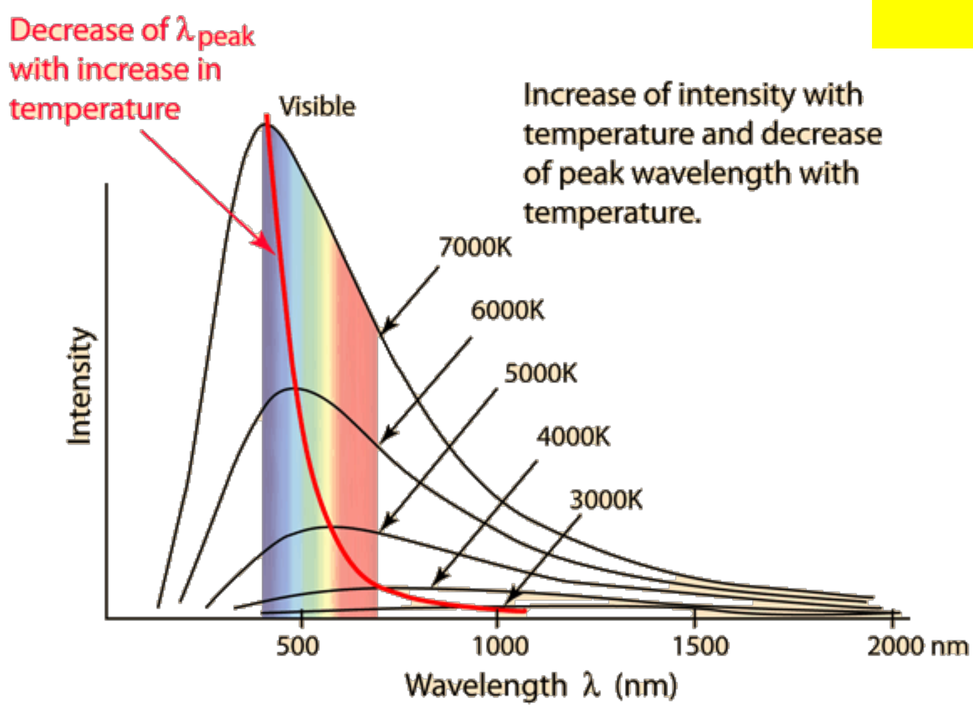
Or

$$\lambda_T = \lambda_o \frac{A(T)}{A(Today)}$$

- The scale factor denotes the distance to a location in space
- The ratio of scale factors shows how much a photon is redshifted from the change in the shape of space.
- Assuming space is only expanding, A(t) will always be an increasing value thus an initial wavelength would only be able to increase which, as expected, would mean the object is moving away from Earth.

Black-Body Spectrum i.e. The Cosmic Microwave Background

Wein's Displacement law: $$\lambda_{max} \propto \frac{1}{Temp}$$



- As the universe expands by a factor of 2, the observed wavelength doubles from Earth's perspective
- From Wein's Displacement law, the Black-bodies cool down by a factor of 2

The Hubble Law

V = velocity
H = Hubble parameter
D = Distance to object

$$V = H_o D$$

- This is used to scale and account for the expansion of the universe.
- Although the Hubble parameter changes over time, this law can be assumed to be linear in small periods of time.
- Note: If a galaxy is twice as far away, GRE students must know it appears to be moving twice as fast away from Earth.

Cosmological Redshift

$$Z(T) = \frac{\lambda_o}{\lambda_T} - 1$$

- This definition uses Z(Today) = 0
- Thus positive values of Z are in the past and negative values denote the future.

Important Note: If a galaxy is at “redshift = 5”,

This means the ratio of the wavelengths is equal to 6.

Doppler vs. Cosmological Redshift

- Note: If the question is asking about the **distances** between galaxies, use cosmological redshift.

- This is because cosmological redshift is due to the expansion of the universe versus the relative motion.