Part G: Traveling Waves

Chapter 112. Waves

Sounds are intimately associated with vibrations. But vibrations stay more or less in one place, while sounds travel from one place to another. According to the definition of sound in Chapter 3, one defining characteristic of sound is that it is “transmitted by … waves.” In fact, in some contexts one might say that sound ***is*** a wave. The fundamental distinction between vibrations and waves is that vibrations involve something that varies in time, while waves involve something that varies in time *and space*. Many (but not all) waves also travel through that space. Here the word *space* is used in the sense of “the collection of all possible positions,” which is much more common in physics than the sense of “up above the atmosphere.”

The category of waves includes an extremely large number of phenomena. In fact, a huge fraction of the field of physics consists of describing nature using two overarching models: waves and particles. There are many properties that apply to all waves, whether the subject is sound waves or quantum mechanical waves. In this chapter, we will define several terms that are used to describe and distinguish between various kinds of waves. But it might help if you read this with one specific type of wave in mind, such as the oldest meaning of the word: ripples or billows on water surfaces.

We start with a definition of the term that includes all the types of waves in physics.

> A wave is a non-uniform, self-sustaining disturbance, usually of a material medium, that extends over a significant span of space and for which the disturbance stays relatively small over long intervals of time.

One way to specify a kind of wave is to specify its medium, e.g., water waves or waves in air. For most waves, the medium is a physical substance, pieces of which are actually displaced just as a vibrating object is displaced. Such waves are classified as **mechanical waves.** There are a few kinds of waves that manage to exist without a material medium, including the very significant **electromagnetic waves** (including radio waves, visible light, and X-rays). Non-mechanical waves will not be discussed much in this book, but they do have many characteristics and much nomenclature in common with mechanical waves.

Disturbance refers to a change away from equilibrium. For vibrations, there is an equilibrium position, but since the wave medium is extended over a large space, it doesn’t make sense to refer to a single position for the whole medium. Instead, the medium has an **equilibrium state**, that is, a way of being that can last indefinitely without change. This usually means that the properties of the medium are the same everywhere, or possibly they vary slowly over large distances, the way that air slowly gets thinner as you move to higher altitudes.

The disturbance can be any change in the medium’s properties. Since the disturbance is non-uniform, in order to get a single numerical measurement, you need to focus on the disturbance of *one small piece of* the medium. For a mechanical wave each piece has its own equilibrium position and vibrates or moves about that position. However, even for mechanical waves, it may be more productive to focus on some other property of the medium, instead of displacement, which also oscillates around an equilibrium value.

When we say that the disturbance stays relatively small over long time intervals, it is in the same sense that the average velocity of a vibrating object is nearly zero (when averaged over a long time), because it never gets far away from its equilibrium position. Similarly, each point in the wave medium remains close to its equilibrium value. In order for the medium to return to the equilibrium state after being disturbed, there must be some **restoring force**. This sounds the same as for a vibration, but the options are more varied for the restoring force of a medium. For example, for a wave on a guitar string, the disturbance is a sideways displacement of the sting. But the guitar string restoring force is the tension, which pulls along the string. This tension removes displacements perpendicular to the string, but somewhat indirectly.

Since the equilibrium state will persist indefinitely, a wave must be started by an external agent. Once the outside force is removed, the medium will return towards equilibrium. But the wave, being self-sustaining, continues on in some way, often by moving the disturbance to different locations in the medium. Only the disturbance can move a long distance, not the pieces of the medium.

Chapter 113. Waves in Time and Space

As with vibrations, graphs are an excellent way to illustrate waves. But there is a catch. Consider, for example, a wave that is a wiggle moving along a string. Even with this relatively simple example, we have three quantities to relate: position along the string, disturbance of the string (a sideways displacement), and time. A basic graph can only show the relationship between two quantities. There are basically two possibilities to handle this: (1) choose a specific time and graph disturbance versus position, or (2) choose a specific position and graph its disturbance versus time.

The first option (disturbance versus position at a specific time) will be referred to as a **snapshot graph** in this book. If the disturbance is a sideways displacement, then a snapshot graph has the advantage of looking almost like an actual photograph of the wave. Figure 113.1(a) shows an example for a wiggle on a string, where the variable x represents position along the string, and Δy represents displacement of the string in a perpendicular direction. For waves that move through the medium, a good addition to a snapshot graph is an arrow indicating the direction that the wave is traveling.

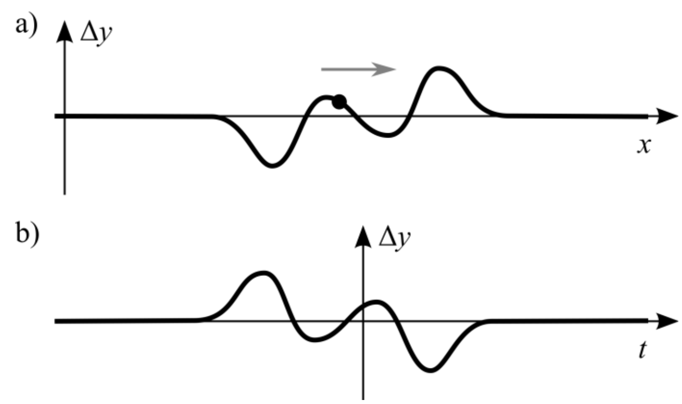

Figure 113.1

Two representations of the same wave. (a) Snapshot at one moment in time, (b) Oscillation of one point in the medium.

A snapshot graph is likely not to be a photorealistic picture, however. The scale for the two axes need not be the same, and in fact the vertical axis is often magnified. For instance, Figure 113.1(a) might well represent a 2 m length of string, with a maximum displacement of only 3 mm. If the disturbance is being described by a property other than displacement, then the graph will not be like a photograph at all. The significance of the term “snapshot” is that it shows a moment in time.

The second option (disturbance versus time at a specific position) will be referred to as a **history graph** in this book. This type of graph is precisely the same as the graphs used for oscillating objects throughout Part C. The only distinction is that the object in question happens to be a piece of a larger medium. Figure 113.1(b) shows an example, which is specifically the displacement history of the black dot in part (a) (perhaps a knot in the string). The time of snapshot (a) is chosen as the time origin for history graph (b); notice that the displacement of the black dot in (a) is the same as the initial displacement in (b).

Even though these two graphs give information about the same wave, the two parts of Figure 113.1 do not look the same. Snapshots and history graphs can look very similar, and sometimes may even appear identical. But they have important differences in what information the graphs contain. Pay careful attention to which type of graph is given each time you see one!

Chapter 114. Wave Types

There are a variety of characteristics that distinguish different kinds of waves from each other. The following paragraphs describe the most important of these for mechanical waves.

Dimensionality

The **dimensionality** of a wave medium describes how many different directions a wave could travel through the medium. A more technical definition of dimensionality is, how many numbers are needed to specify a location in the medium. Dimensionality is really a property of a wave medium, but it is sometimes used to describe a wave in that medium. A wave in a taught rope is a one-dimensional wave, since the rope is a one-dimensional medium.

This does not mean that the rope itself is one-dimensional—it has thickness as well as length. It also does not mean that the motion in the wave is constrained to one physical dimension—it has sideways displacements. But a position along the rope can be specified by a single number, such as the distance from the end. This is directly linked to the fact that a wave in the rope can only move in one dimension, along the rope. A similar thing can be said for a narrow tube full of air. An audible sound inside the tube does not bounce around off the tube walls; it can only move back and forth along the tube. If someone shouts into one end of a very long tube, the position of the shout as it travels down the tube can be described by just one number.

Position on the surface of a pond requires two numbers, for example distance along the length and width of the pond, or distance away from a center and an angle for direction. Thus, the surface of a pond is a two-dimensional wave medium, even though the water extends into its depth. A ripple can move in any direction along the surface, but it can't move up into the air or down under the water. We will see shortly that the disturbance involved in a water surface wave does extend into the depth of the water. But that disturbance in the depth is synchronized with the disturbance at the surface, so that it does not introduce a new, separate dimension.

Sound traveling through air is a three-dimensional wave. To recognize 3D waves, notice that for a source on the ground, the wave travels (1) away from the source, both (2) in various directions along the ground (the **azimuthal angle**) and also (3) at various angles away from the ground (the **elevation angle**). If a medium fills a large volume and waves in it can travel in any direction at all, then that is a three-dimensional wave medium.

Traveling & Standing

When there are few boundaries to the medium, waves are usually encountered moving from one place to another, as suggested by the definition of sound in Chapter 3. These are **traveling waves.** Sometimes the wave only disturbs a small region within the medium, so that one can track how that disturbed region moves through the medium. Other times, the wave is more extended, even filling all of the medium. But even then, it's possible to locate and track an extreme of the disturbance that makes the wave. For example, waves on the ocean fill huge regions, but you can still track the motion of a wave crest. Traveling waves are the focus of this Part of the book.

However, all types of waves can take a second form called **standing waves,** especially when the waves are constrained between boundaries of the medium. In standing waves, although the medium moves in a periodic way, the locations of extremes of the disturbance do not move through the medium. Standing waves will be considered separately in Part I.

Motion of the Medium

For a mechanical wave, the pieces of the medium move, often in a repetitive way. It is very important to keep in mind that the motion of a traveling wave itself is a very different thing from the motion of the pieces of the medium. In particular, the wave (that is, the disturbance in the medium) can travel long distances, but the pieces of the medium themselves never stray far from their equilibrium positions. When "the wave" is performed in a sports stadium, the fans hardly leave their seats, and yet the disturbance travels around the whole stadium.

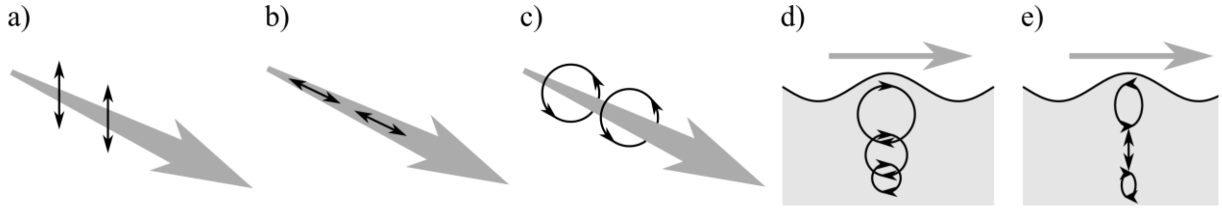

Figure 114.1

Waves differing by motion of the medium. Gray arrows show direction of wave travel, black arrows show motion of pieces of medium. Examples (a–c) are in perspective, coming out of page. Waves in (d) and (e) are associated with surface of a substance, shown in light gray.
(a) transverse wave, (b) longitudinal wave, (c) circular wave, (d) water surface wave, (e) Raleigh surface wave.

As one piece of the medium moves, it traces out some shape. Figure 114.1 shows five examples in traveling waves, but there are other kinds as well. The alternatives for that traced shape are indicators of different kinds of waves. It is also significant how the traced shape relates to the direction of wave motion (for traveling waves). Notice that the circular shapes in Figure 114.1(c) and (d) are oriented differently to the gray arrow. Even for standing waves, there can be an axis in the wave that serves the same function as the direction of travel. Some kinds of waves can only happen in certain dimensions: mechanical circular waves (Figure 114.1(c)) can only happen in one-dimensional media. Some kinds can only occur in certain types of media: Raleigh waves occur only in solids. The main point is that there is quite a variety of possibilities.

The physical extent of moving material does not always match the dimensionality of the wave. The surface waves in Figure 114.1(d) and (e) extend into the depth of the substance. However, the disturbance can only travel across the surface, making these two-dimensional waves. The subsurface motion is linked to the surface motion, moving with it.

Despite this variety, elementary books on waves usually focus on just the first two examples from Figure 114.1. In both cases, every piece of the medium oscillates along a line, just like an object oscillating due to a restoring force. In **transverse waves,** the medium motion is perpendicular to the wave motion. The classic example is a wave on a string. In **longitudinal waves,** the medium motion is along the same line as the wave motion. These are less frequently visible; a stretched slinky that is required to stay straight can make an example. Mechanical longitudinal waves are also called **compression** waves, because they must result in some of the medium being compressed while other parts are stretched. These two wave types will be the focus of the current text, because the usual sort of sound is a longitudinal wave, and transverse waves provide a more visually accessible example of wave behavior. Transverse waves are the ones for which a snapshot graph resembles a photograph of the wave.

In some types of wave, the *direction* of medium motion can change without changing the *type* of medium motion. The different choices are then called **polarizations.** For instance, circular waves might have the medium moving either clockwise or counterclockwise. As another example, in transverse waves there are many motion axes that are perpendicular to the wave motion. For longitudinal waves, however, there is only one axis that the medium can move, so the polarization idea does not apply.

Extent

Another important characteristic is the duration of the disturbance. **Duration** refers to the time from when a wave starts disturbing a particular place in the medium, to when it stops disturbing *that same place*. Notice that for a traveling wave, this is not the same thing as how long the wave lasts. Suppose that a speaker issues a beep sound for 0.25 s, as illustrated in Figure 114.2. You are standing 680 m away, and 2 s after the speaker made the sound, you hear the beep for 0.25 s. The duration of that sound is 0.25 s, even though the wave has been in existence for at least 2 s (while traveling from the speaker to you) and will probably continue to exist for a while after that.

If a snapshot were taken of a traveling wave, recording the disturbances present at a single moment in time, the **length** of the wave is the distance *along the direction of motion* over which the disturbance is present. Notice that this may not be the same thing as the size of the wave. Suppose that the speaker above is sending out sound in all directions. The length of the sound wave would be about 85 m, as shown in Figure 114.2, and would stay constant as the sound travels. But by the time the sound reaches you, it has also traveled 680 m in the opposite direction, so that the "size" of the sound could be said to be $2 \times 680\ \text{m} = 1360\ \text{m}$ and getting bigger.

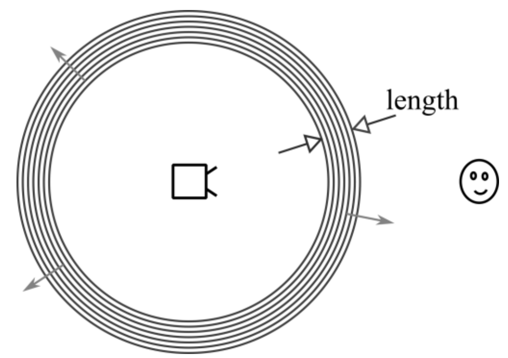

Figure 114.2
Brief sound shortly after leaving a speaker.

For traveling waves, the duration Δt is directly linked to the length L of the wave by how fast the wave is traveling. Since it travels one length in the time of a duration, the wave has a speed is

$$s = \frac{L}{\Delta t} \quad . \tag{114.1}$$

Waves at a beach, which seem to never stop, are called **continuous** waves. This also implies that the ripples continue indefinitely out into the ocean. Of course, nothing can truly last forever or extend over an infinite distance. Nevertheless, this is often a useful model. (It you have read Chapter 23, this is similar to the way simple harmonic motion is useful, even though no vibration lasts forever.)

At the other extreme, a disturbance on a stretched rope can consist of only a single bump, which is called a **wave pulse**. Intermediate cases are sometimes called **wave packets**. These are illustrated in Figure 114.3. Although these are presented as snapshots of a moment in time, the same shapes could be displacement-versus-time graphs as the wave passes through a single point in the medium. Notice that it may not be possible to get a precise value for either the duration or the length of a wave, since they fade out at the ends rather than ending abruptly.

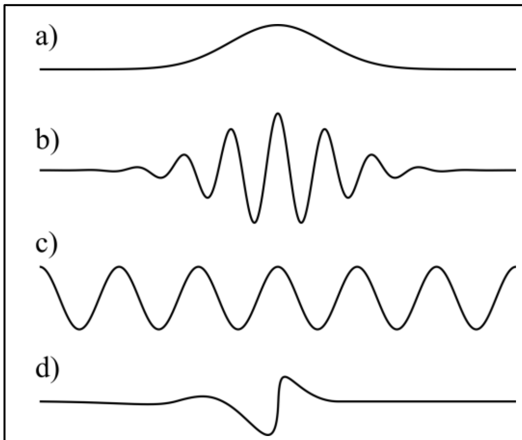

Figure 114.3
Snapshots of waves with different extents. (a) Wave pulse, (b) Wave packet, and (c) Continuous wave. (d) Shock wave graph, compression versus position.

One specific example is a **shock wave,** which is a three-dimensional, traveling, compression wave made of a very short wave packet. Figure 114.3(d) shows an example, although since it is a compression wave, the line drawn is a graph of compression rather than a shape that you could see. The thing that makes this a shock wave is the place where the graph changes value very abruptly. See Chapter 124 for more about graphing compression waves.

Chapter 115. Dispersion

For vibrations, Section 20c shows that a few simple rules led to the "Circle of Physics" and a very specific shape for vibration graphs. Waves have no such restriction—wiggle one end of a taught rope however you like, and a wide variety of shapes will travel along it. Figure 115.1 shows three different wave packets, all traveling in the same direction. Packets (b) and (c) are the same shape except for the fact that (c) is more spread out.

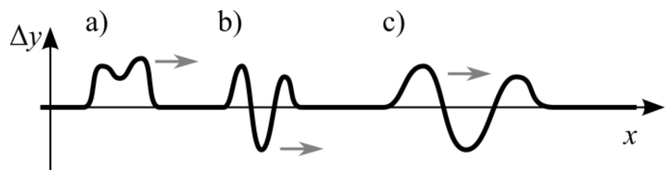

Figure 115.1
Several differently shaped wave packets on string.

Do differently shaped waves behave differently? In the most general case, yes, they can. For instance, if (b) and (c) in Figure 115.1 represented ripples on the surface of deep water, we would find that ripple (b) moves faster, eventually catching up to (c). When wave shape influences wave speed, the medium is described as **dispersive.** The name relates to the fact that if there are two wave packets near each other and traveling at different speeds, then they will eventually get further apart from each other. That is, they will disperse.[37]

For almost any type of wave, if the disturbances get too large, then the shape of the wave affects its behavior. This is related mathematically to the fact that if any real spring is deformed too far, it starts to disobey Hooke's Law. However, there are very many situations in which wave shape does *not* affect wave speed. Instead, the wave speed is determined by the medium alone. This is why Chapter 7 was able to refer to the speed of sound before this book even considered vibrations, let alone waves. Describing exactly what sort of restoring force results in a shape-independent wave speed is complicated. Suffice it to say that such media are fairly common and are called **non-dispersive media.** This book is almost entirely concerned with non-dispersive media.

In a dispersive medium, even a single wave packet may change shape as it moves, since it has different parts with different shapes. The inverse of this fact gives another important property of non-dispersive media: a wave retains the same shape as it moves through a non-dispersive medium. This has significant implications for the graphs we use to represent waves. Figure 113.1 was drawn assuming a non-dispersive medium. Since the same wave shape is passing through every point in the medium, the history graph for any location would look just the same as part (b), except shifted in time depending on when the wave pulse arrives. Although a history graph is technically associated with a single, specific position in the medium, for a non-dispersive medium a history graph can illustrate what occurs at all positions.

The unchanging wave shape also allows a more specific link between the snapshot graph and the history graph. At the time that snapshot Figure 113.1(a) was taken, the tallest bump had already passed through the black dot. That is why the tallest bump appears in the history graph (b) in the past (time before $t = 0$ s). Immediately after snapshot (a), the wave will move to the right, and the black dot will move towards positive Δy. That is what is shown near $t = 0$ s in part (b). As time moves forward, the black dot sees parts of the wave that were at smaller and smaller x values in snapshot (a). The end result is that, for a wave moving left-to-right in a snapshot, the history graph looks like the snapshot flipped left-to-right.

Every day we all experience a fact about sound waves that might seem surprising on second thought: they can pass right through each other without any disruption. There is usually a myriad of sounds around us, crisscrossing as they move through the air, and yet no sound is ever distorted by the presence of other sounds. This would be quite amazing if sounds traveled as small objects that might run into each other. But it is standard behavior for non-dispersive media, which usually have the property called being **linear.** When two waves arrive at the same location in a medium, they do combine to give the disturbance a different shape than would occur with either wave alone. However, the multiple wave velocities (magnitude and direction) are not affected, and each of the two can continue on as if the other were not there.

[37] There are other senses in which different speeds cause different waves to disperse. The earliest use of the term has to do with making rainbows from prisms.

Chapter 116. Wave Speed

For a non-dispersive medium, it is always possible to predict the wave speed based only on properties of the undisturbed medium. In solids and liquids, a sufficiently accurate model can often be found based simply on what the material is, as is done in the tables referenced in Section 7a. But for gasses, and also for greater accuracy in liquids and solids, it is necessary to consider other properties of the medium. We see an example of this in Section 7b, with the temperature dependence of the speed of sound in air.

In general, to determine a wave speed we need to know something about the restoring force in the medium, and something about how "heavy" the medium is. In fact, the formula giving the speed of wave travel always has the same general pattern, a pattern very similar to Eq. 27.2 for vibration frequency. It is always the square root of a fraction, with a parameter describing the restoring force in the top and a parameter describing the "massiveness" in the bottom,

$$\text{wave speed} = \sqrt{\frac{\text{restoring force}}{\text{massiveness}}} \quad . \tag{116.1}$$

To determine how this pattern turns into a formula for any specific medium, it may be necessary to learn a good deal about what provides the restoring force, and how to express the massiveness.

When learning about different examples of wave speed formulae, it's valuable to use a tool called **dimensional analysis.** This is an important way to check an equation for completeness (and sometimes even to discover an equation). It simply means ensuring that the units on both sides of the equation match, after any necessary algebraic manipulations.

Dimensional analysis can never prove that an equation is correct. For instance, it can't detect whether there should be a numerical multiplier, like 2 or π. But dimensional analysis can rule out incorrect equations, and even suggest what type of quantity might be missing. And rather often, it turns out that the simplest possibility is in fact the correct option.

Chapter 117. Volume Mass Density

Everyone knows that steel is "heavier," or "more massive," than wood. But what does that mean? It is entirely possible for a large log to weigh more than a small steel ball. Of course, what is meant is that given equal size pieces (unit volumes, one might say), the steel piece is heavier.

When describing a medium through which a wave might travel, a "heavier" medium is harder to move, which results in a slower wave speed. But the medium might extend much farther than the length of the wave, so again this is not referring to the mass of the entire medium, but instead the mass of a standard amount of the medium. For three-dimensional waves (such as sound), the standard amount is a volume.

To define heaviness of a medium quantitatively, we need to separate the closely allied concepts of weight and mass. Weight is the force with which gravity pulls an object down, but we don't really care about gravity in the current context. What is important here is the more intrinsic property of mass, which describes how much material, or how much "stuff," an object has. That description is sufficient here, but for more details see Chapter 15 about mass and Section 18b about the connection to weight.

The SI root unit of mass is the gram, with the symbol g. Other units of mass can be obtained by adding prefixes to gram in the usual way. However, the SI base unit of mass is the kilogram (kg), which means that the kilogram is used to build named derived units. This is the only place in SI where the base and root units are different, which can be a source of difficulty. Usually, the safest calculation method is to use kilograms in all calculations. Something with a mass of 1 kg weighs a little more than two pounds.

For a particular material or substance, the mass of different size chunks of that substance will be proportional to the volume of the chunks. Taking the ratio, as usual, we get

$$\rho = \frac{m}{V} \quad , \tag{117.1}$$

where V is the volume of an arbitrarily sized chunk, m is the corresponding mass (that is, of the same chunk), and ρ is the **volumetric mass density** of the substance, almost always shortened to just **density**. It is traditionally represented by the Greek letter rho (ρ, which is *not* a Roman p), and has an SI unit of $\mathrm{kg/m^3}$. Why not just use the initial letter d? There are some references which make that choice, but d has so many other common uses that this tends to lead to confusion.

It is quite a good model to say that volume density is an intrinsic property of solids and liquids under most circumstances. The density can vary a little with conditions, such as temperature, but that often can be ignored. But when sounds pass through solids or liquids, tiny variations of the density are very important. Gasses can vary in density by a much larger fraction.

Chapter 118. Linear Mass Density

For a one-dimensional wave, for example one passing along a rope or cable, choosing different sized chunks of the medium takes a different form than it did for a three-dimensional wave. Imagine a thin steel cable, and a fiber rope that is thick enough so that a 1 m length of the rope has the same mass as a 1 m length of the cable. The steel is more dense than the fiber by volume, so the fiber rope will need to be thicker (larger radius) than the steel cable in order to make this so. But a wave passing along either the rope or the cable has to displace the same mass as it passes through that meter. How much the mass is compressed radially makes no difference. It is how the mass is spread along the length of the medium that is important.

For any given one-dimensional medium, the mass of a segment of the medium is proportional to the length of the segment. Taking the ratio as usual, we get

$$\mu = \frac{m}{L} \quad , \tag{118.1}$$

where L is the length of an arbitrarily sized segment, m is the mass of that same segment, and μ is the **linear mass density** of the medium. The algebraic variable used in this book is the Greek letter mu (*not* a Roman "*u*"), the same letter that is used for the metric prefix micro-. Another common variable choice is Greek lambda λ.

Notice in particular that the L in Eq. 118.1 does not have to be the same as the length of the rope that the wave travels down. As is usual with proportions, all that is required is that the m and L in Eq. 118.1 correspond to each other. The resulting μ then describes any sample of the same rope.

The SI basic unit for linear mass density is kg/m. However, a cable that had a mass of 1 kilogram for each meter would be quite impressive, suitable perhaps for construction or for anchoring large ships. It is more common to express linear densities with the unit g/m. This is the source of a common difficulty, when in a calculation the linear mass density is combined with other quantities in SI units. Be on the lookout for this, since the unit will typically need to be converted from grams to kilograms.

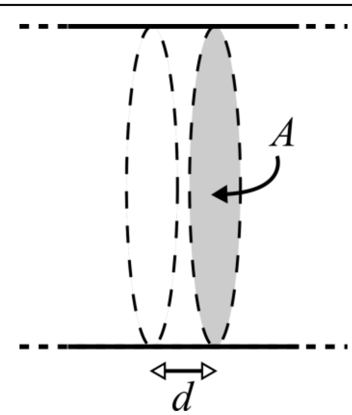

Figure 118.1
Close view of interior of a tube, with volume between two disks highlighted.

Since linear mass density depends on both the material and the shape, it can't be looked up in a reference the way volume density can be. But we can calculate it. Figure 118.1 focuses in on a thin cylindrical slice from some 1D medium, with cross-sectional area A, and a small thickness d measured along the axis of the medium. By calculating the mass m in the cylinder in two different ways, the linear mass density μ can be related to the volumetric density ρ by

$$\mu d = m = \rho V = \rho A d \quad , \tag{118.2}$$

$$\mu = \rho A \quad , \tag{118.3}$$

where V is the volume of the cylindrical slice. One outcome is that in cases where the area is constant along the length of the medium, the two densities are proportional.

Chapter 119. Wave Speed in a String

We now consider the speed of a transverse wave on a string, rope, or similar long, thin medium. How this speed depends on the characteristics of the string can be derived with college-level physics, but it is beyond the level of this book. Nevertheless, Chapters 116 and 118 have laid the groundwork for us to make a good guess.

To put a wave on a string, there needs to be an equilibrium state resulting from a restoring force. If the string is just lying on a table, there is nothing to keep it in any particular shape. So, while it may be possible to shake one end and make the rest of the string move, that does not qualify as a wave. Since strings are floppy, the only option is a force pulling the string ends away from each other. The resulting tightness in the string, a force with which every piece of the string is pulling on its neighbors, is called a **tension force.** The tension can be supplied from any other sort of force pulling on the string ends: a hand, gravity pulling down on a mass tied to the string, a spring, etc. But the force we need to focus on here is the tension force that is actually in and throughout the string.

Since the size of a one-dimensional medium can only be described with a length, the appropriate way to indicate massiveness is the linear mass density μ. Following the model in Eq. 116.1, and using the variables that seem to be available, we can guess that the speed of a transverse wave on a string might be

$$s_{\text{rope}} = \sqrt{\frac{F_T}{\mu}} \quad , \tag{119.1}$$

where F_T is the tension force. Dimensional analysis can check whether this equation is sensible, by making the "calculation"

$$\sqrt{\frac{\text{N}}{\text{kg/m}}} = \sqrt{\text{N} \cdot \frac{\text{m}}{\text{kg}}} = \sqrt{\left(\text{kg}\frac{\text{m}}{\text{s}^2}\right) \cdot \frac{\text{m}}{\text{kg}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{m}}{\text{s}} \quad . \tag{119.2}$$

In this case, not only is dimensional analysis satisfied, but Eq. 119.1 turns out to be exactly correct.

This model assumes that the tension is the same throughout the string or rope. But because the string or rope has some mass, gravity will pull down on its parts, sometimes marring this idealized picture. A hanging rope has a higher tension at the top than the bottom, because the top has to support the bottom. A rope stretched horizontally will sag, so that the tension is higher in near the ends of the rope to support the middle. (The equilibrium shape also is not a straight line, but that is not a problem itself—any equilibrium shape will do.) For this book, we'll assume that the gravitational force on the string is negligibly small compared to the string tension, and therefore the tension is equal throughout the string. For stringed musical instruments, for example, this is an excellent model.

This model also assumes that the tension is the only restoring force. Some "strings," such as those for pianos or guitars, are fairly stiff, and will straighten themselves even without any tension. We can guess that this model will not work perfectly in those cases. Indeed, that is the case, with some significant consequences in the case of tuning pianos. However, it turns out that the stiffness also leads to a small degree of dispersion in the medium, so that is beyond the scope of this book.

Chapter 120. Sound Speed in a Gas

A gas (such as air) is a fluid, so Chapter 133 Sound Speed in a Fluid will technically apply to gasses. But, as mentioned there, this is not the most useful approach. The details of further analyzing Eq. 133.1, belonging to the physics field of thermodynamics, are beyond the scope of this book. You won't need to read Chapter 133 to understand this chapter.

The end result is that the speed of sound in any specific gas depends only on the temperature. The full equation is

$$s_{\text{gas}} = \sqrt{\frac{\gamma RT}{M}} \quad , \tag{120.1}$$

which has several new variables that need to be explained.

- T is the temperature of the gas, but it must be measured in units that set 0 to be at **absolute zero**, the coldest possible temperature. The usual choice is the unit kelvin (K for short). To convert temperatures in degrees Celsius into kelvin, you can use the formula $T[\text{K}] = T[°\text{C}] + 273.15$ K.
- M is the mass of one molecule of the gas, measured in "atomic mass units" or amu. These are mass units in which a single hydrogen atom, which is the smallest possible molecule, has a mass of 1.008 amu. (Warning: Hydrogen gas is normally made of molecules comprising two atoms, not one.) For a mixed gas like air, containing several different types of molecules, M is an "average" molecular mass. This book won't go into how to find this sort of average.
- R is the same for all gasses, and is simply named the **universal gas constant**, with a value of $8314\ (\text{amu/K})(\text{m/s})^2$. The crazy units are chosen to be compatible with the other units used here.
- γ (the Greek letter gamma) is a number that ranges between 1 and $\frac{5}{3}$, depending on the gas. This book won't go into any details, except to say that the value for air is $\gamma_{\text{air}} = 1.4$.

Clearly, there is a lot of physics detail in Eq. 120.1. But from the perspective of this book, those details are not as important as the following general observations.

Equation 120.1 can still be viewed as having the pattern from Eq. 116.1. The "massiveness" part is hopefully clear, even if the required units are not familiar. The "restoring force" part comes from the fact that at higher temperatures, molecules have more energy, and therefore are more resistant to outside influence.

Hotter gasses result in a higher speed of sound. Also, smaller molecules (for example, helium) result in a higher speed of sound.

Figure 120.1 shows a graph of how the speed of sound in air depends on temperature (the solid line). The origin of the temperature axis is absolute zero. The temperatures at which water freezes and boils are also indicated. Although the true relationship is a curve, we can see that between freezing and boiling a straight line is a very good approximation. The dotted line is the approximation from Section 7b

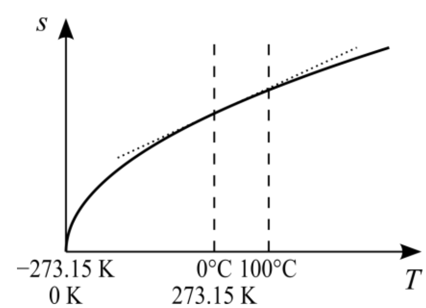

Figure 120.1

Speed of sound in air, dependency on temperature.

$$s_{\text{sound}} = 331.3\,\frac{\text{m}}{\text{s}} + \left(0.6\,\frac{\text{m}}{\text{s}}\right) T[°\text{C}] \quad . \tag{7.1}$$

Although this model is less exact than Eq. 120.1, it can be quite a bit easier to use.

Chapter 121. Periodic Waves

Section 20c shows that given a particular system (an object with a restoring force, a quite common situation), physical relationships require the system to vibrate in a very specific and periodic way. Waves are not constrained like this. By shaking a hanging rope it is easy to find that a transverse wave can take any shape. It's true that very large displacements from equilibrium are likely to break some of the simplifying assumptions that we are making in this book. But they are not disallowed by any general rule about waves.

Nevertheless, periodic waves are of special interest, because they are related to the periodic vibrations which make musical sounds. Figure 121.1 shows a periodic wave. Since the history graph (b) is an oscillation, is still has all the properties of an oscillation, such as **period** T and **frequency** f . These parameters now adhere to the wave itself.

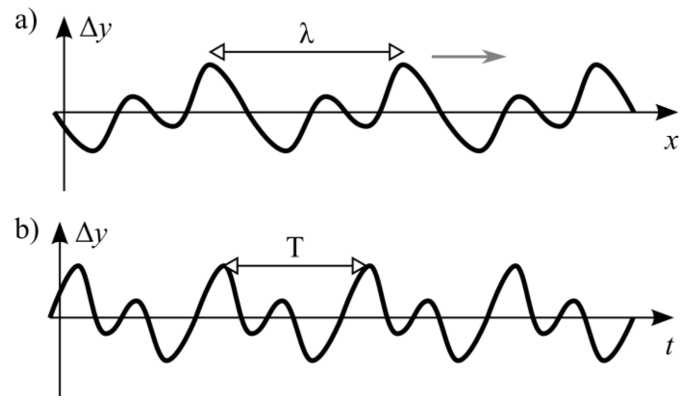

Figure 121.1

Two representations of same wave. (a) Snapshot at one moment in time. (b) Oscillation of one point in the medium.

The snapshot graph in Figure 121.1(a) also has a repeating shape. Here the horizontal axis represents position, and the length of one cycle is called the **wavelength** of the wave. It is symbolized by the Greek letter λ. While the root unit for wavelength is the meter (just like any other length), it is often helpful to consider wavelength to have the unit $\mathrm{m/cycle}$. This is similar to unit for period; see Chapter 10 for details. Avoid confusion with the terms: the wavelength is *not* the same as the length of the wave.

If Figure 121.1(a) were a movie, you could watch it just long enough for the wave to advance to the right by one wavelength. A snapshot at that final time would be identical to the initial snapshot. The time required for this is precisely one period, since every point in the medium completes one cycle of motion in that time. Since the wave travels one wavelength in the time of one period, the wave speed is

$$s = \frac{\lambda}{T} \quad , \tag{121.1}$$

$$s = f\lambda \quad . \tag{121.2}$$

Once again, recall that the wave speed here is something completely different from the speed of a particle in the medium. After all, one speed is a constant, while the other speed is oscillating.

Equations 121.1 and 121.2 are a very fundamental relationship for waves, or even anything somewhat like a wave. For instance, suppose that you are waiting for a train to pass at a level road crossing. The train is rather like a wave, repeating every car length. If you estimate that one train car is 16 m long, and that you see one pass every 0.8 s, then you can calculate the speed of the train with

$$s = \frac{\lambda}{T} = \frac{16\ \mathrm{m}}{0.8\ \mathrm{s}} = 20\,\frac{\mathrm{m}}{\mathrm{s}} \quad . \tag{121.3}$$

Chapter 122. Sinusoidal Waves

If a wave has the shape of a sinusoid, then it is not only periodic, but it also inherits two other concepts from sinusoidal oscillations: amplitude and phase. The connection comes from the fact that as a sinusoidal wave passes through its medium, each small piece of the medium moves with simple harmonic motion.

For example, Figure 122.1(a) shows a snapshot graph of a sinusoidal wave at the specific time $t = 0$. As the wave moves, the black dot will oscillate up and down with a displacement versus time shown in Figure 122.1(b).

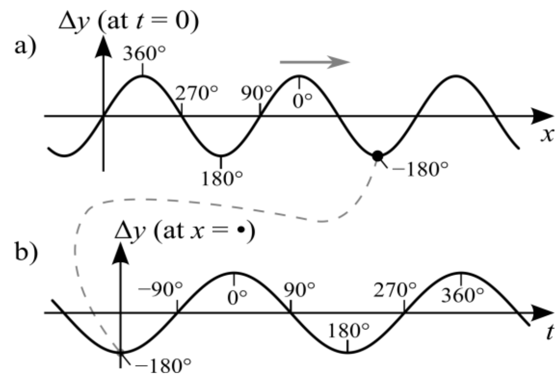

Figure 122.1
Snapshot and history graph for same sinusoidal wave. Snapshot is for $t = 0$, and history graph is for position marked with a black dot.

The **amplitude** y_m (as well as the **peak-to-peak amplitude** y_{pp}) is straightforward to define. The amplitude of the oscillation of one piece of the medium is identical to the maximum displacement found in the wave. The only change is that since we are using the variable x for position along the wave, the amplitude variable is something else. As with SHM, the amplitude of a wave can be changed with no effect on the other parameters of a wave.

Phase requires a little more thought. Recall that phase is not a single number, but a variable that indicates location on a sinusoidal shape. On the history graph for the black dot, in Figure 122.1(b), various phases have been labeled in the usual way for oscillations: 0° is at a peak (because this book focuses on the cosine function), phase increases with time at the rate of 360° per period, and the oscillation has (for this choice of the placement of 0°) an initial phase of $-180°$.

Now consider the snapshot graph Figure 122.1(a). At that time ($t = 0$) the phase at the dot is $\phi = -180°$. In the near future, the phase there will *increase* to $\phi = 0°$. This means that in the snapshot graph, phase must increase *in the opposite direction from the direction of wave travel.* This is the same effect that causes the history graph in Figure 113.1 to look different from the snapshot in that Figure. For a sinusoidal wave, reversing the graph along the horizontal axis does not change the shape, but it does reverse the phase labels.

Chapter 123. Sinusoidal Wave Function

Building from the SHM equation in Chapter 22, a similar equation can be developed that describes a sinusoidal wave. As in that case, although such an equation will contain all the information about the wave, that does not necessarily mean that it is the most useful tool for answering all questions. The function is mainly useful in mathematically relating position and time to the wave displacement or other disturbance.

Each piece of the medium is oscillating with the same period and amplitude, but the initial phase for each piece is different. By updating the variables in Eq. 22.3, these oscillations can be described by

$$\Delta y = y_m \cos\left(\frac{360°}{T} t + \phi_i\right) \quad . \qquad (123.1)$$

Figure 122.1(a) shows the time $t = 0$, so the phases there are all initial phases ϕ_i of the respective pieces.

Just as an oscillation has phase differences proportional to time differences, in a snapshot graph phase differences are proportional to differences in position. Analogous to Eqs. 21.2 and 22.2, we can write in relation to Figure 122.1(a)

$$\frac{360°}{\lambda} = \frac{-\Delta\phi}{\Delta x} = \frac{-(\phi_i - \phi_0)}{x - 0} \quad , \qquad (123.2)$$

where ϕ_i is the initial phase for the position x. The negative is added to represent that phase increases as position decreases. The symbol ϕ_0 is the special phase at $t = 0$ and $x = 0$, which is called the **phase constant** of the wave, and which for Figure 122.1 is $\phi_0 = 450°$ (a **reduced phase constant** of 90°). Notice that while "phase constant" and "initial phase" were synonyms for oscillations, for waves they mean something different.

By solving Eq. 123.2 for ϕ_i and substituting that into Eq. 123.1, we arrive at the wave function

$$\Delta y = y_m \cos\left(\frac{360°}{T}t - \frac{360°}{\lambda}x + \phi_0\right) \quad . \tag{123.3}$$

Here we see in algebraic form the fact that a wave is a disturbance that depends on both position and time. To be clear, even though we used snapshot graph Figure 122.1(a) to find this equation, the phase constant ϕ_0 is a property of the entire wave, along with y_m, T, and λ.

Chapter 124. Longitudinal Compression

Longitudinal waves are less easy to visualize than transverse waves, mostly because even when you can see a compression wave (for instance in a slinky), it is hard to see where the equilibrium positions were. For a transverse wave on a rope, the equilibrium state is easy to remember (a straight line from end to end), so it is easy to see the displacements. Figure 124.1 shows a slinky both (a) in the equilibrium state, and (b)

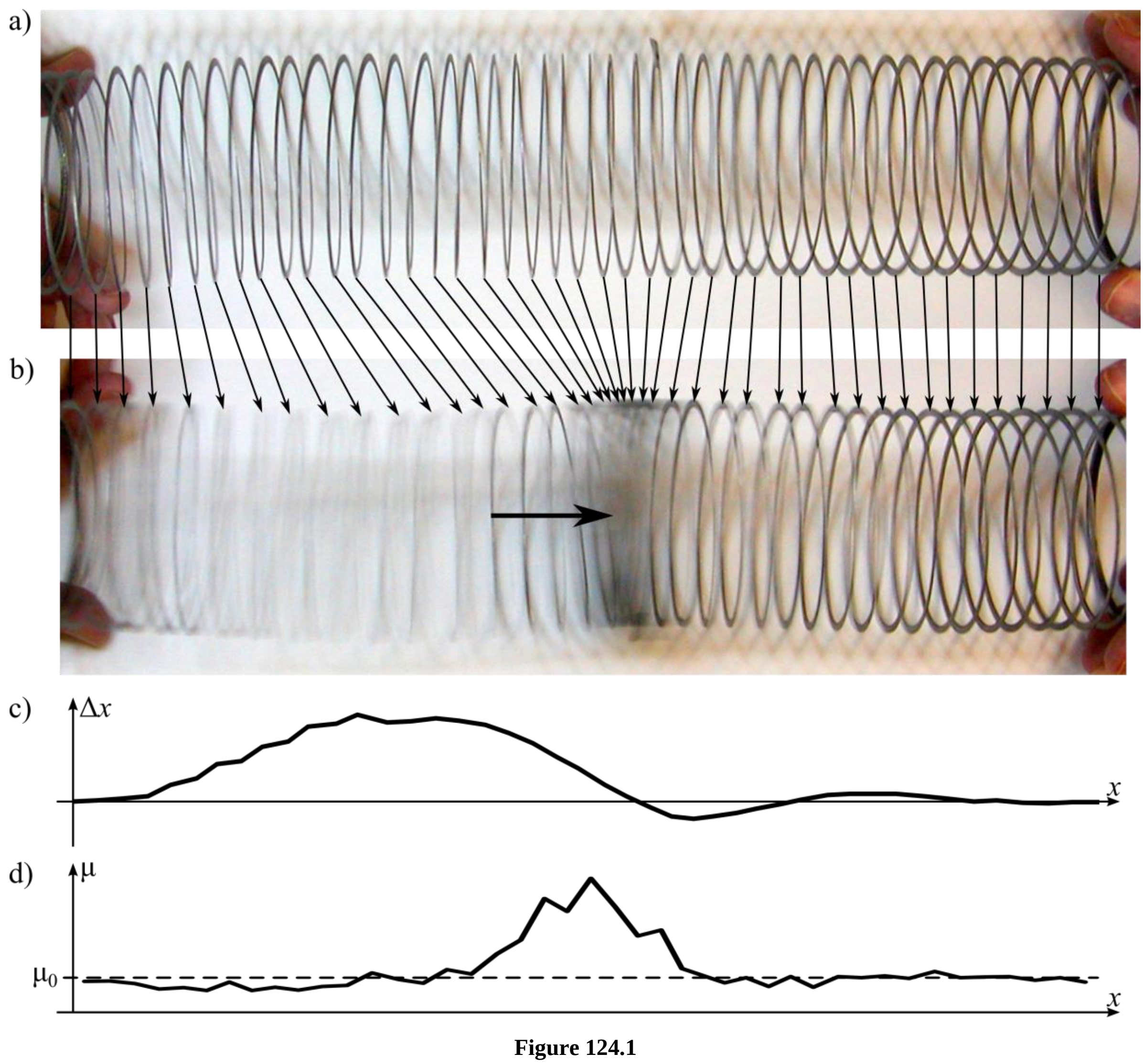

Figure 124.1

Compression pulse wave passing along slinky. (a) Slinky in equilibrium state. (b) Snapshot of pulse passing along slinky; thin arrows connect equilibrium position to displaced position. (c) Longitudinal displacement versus equilibrium position. (d) Linear density versus equilibrium position.

with a compression pulse wave passing along it left to right. The displacement can be seen from the arrows connecting the equilibrium to displaced positions.

To make this more manageable, Figure 124.1(c) graphs the displacement of each loop versus the equilibrium position of that loop. The usual horizontal x-axis has been chosen. Each loop is displaced longitudinally, along the same axis as the wave is traveling, so Δx is the appropriate variable for the displacement. This graph is the longitudinal equivalent of a snapshot graph. Notice that Δx here means the position change between equilibrium and with the wave; it does *not* mean the length of one loop.

But the displacement graph doesn't capture what might be the most obvious feature of Figure 124.1(b): that the loops are compressed together in the middle, and stretched apart on the left. To reveal this **compression** and **rarefaction**, the linear mass density in Figure 124.1(b) is graphed in Figure 124.1(d). Each loop has the same mass, but has a different length while the wave pulse is passing through, and thus has a variable density. On the right, the loops are still in the equilibrium state, since the pulse has not arrived there yet. In equilibrium, every slinky loop takes up the same length, so that the linear mass density is the same value everywhere, indicated by μ_0 on the graph.

You may notice that the peak in graph Figure 124.1(d) doesn't quite line up with the most dense part of photo (b). That's because the density has been graphed versus equilibrium position, rather than disturbed position. Even though the resulting graph don't line up with the picture, this is the less confusing option. If a movie were made showing the wave propagation, points on the graph would move only vertically, rather than both vertically and horizontally.

The compression in the middle of Figure 124.1(d) is much more obvious than the rarefaction on the left. There is a sense in which the rarefaction and compression should exactly cancel each other: their combination in photo (b) yields a total length that is exactly the same as in photo (a). But the mathematical definition for linear density, and also the choice of equilibrium position for the horizontal axis, both tend to emphasize compression over rarefaction in graphs. This matches a natural human tendency to focus on increased density more than decreased density. These waves are called compression waves far more often than rarefaction waves, even though both aspects are present.

Thus, for longitudinal / compression waves we have two different representations, displacement and density. Figure 124.2 shows a similar presentation for a sound wave passing through an open-ended pipe. By constraining the sound to a tube, a three-dimensional wave is effectively converted into a one-dimensional wave. We are assuming that any thin disk-shaped slice of air that is a cross section of the tube moves as a unit, with all the bits of air in the disk moving in concert. The sound could be traveling either left-to-right or right-to-left; the direction of travel doesn't influence the graphs.

Some slices of air are marked with dots, so that in Figure 124.2(b) you can see their displacement from the equilibrium positions in part (a). Part (c) graphs the longitudinal displacements, which vary sinusoidally. The direction of positive displacement is determined by the choice of axis direction, not by the direction of wave travel.

Part (d) shows the change in linear density. With the wave there, many slices occupy a length along the tube that's different from

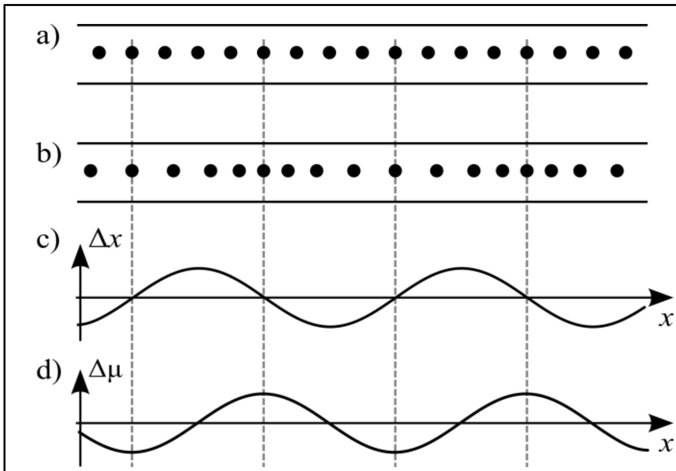

Figure 124.2

(a) Air at equilibrium in pipe. Dots mark a few parcels of air. (b) Sinusoidal sound passing through pipe. (c) Displacement of the air, as function of equilibrium position. (d) Change in air density due to the sound.

without the wave, changing their linear density according to Eq. 118.1. The density in equilibrium is not zero, of course, but (d) only graphs *changes* away from equilibrium. The vertical dashed gray lines show places where the slices of air either have been pushed together, thus increasing the density to a maximum (a compression), or have been spread apart, thus decreasing the density to a minimum (a rarefaction). Here we see the tendency of sinusoidal variation in one variable to result in sinusoidal variations in other variables: the linear density is also sinusoidal.[38]

For these sinusoidal waves, the idea of amplitude is subtly different from before. Neither the position nor the density are zero at equilibrium. The equilibrium *x* even differs for every different part of the medium. The Δ has more significance, because it is the changes in these quantities that varies sinusoidally. Nevertheless, to simplify notation, the amplitudes are symbolized without the Δ. The maximum value of displacement Δx is denoted x_m and the maximum value of density $\Delta\mu$ is denoted μ_m.

One important difference between these graphs is that they are shifted sideways from each other by one-quarter cycle. The density is most changed, either up or down, where the displacement is zero (at the gray dashed lines). The extremes of displacement occur where the density is unchanged.

Chapter 125. Drawing Waves

125a. Drawing 2D Waves

In Chapter 113 we faced the problem of how to show a relationship between three wave variables (displacement, position, and time) on graphs with only two axes. The solution was to have both history graphs and snapshot graphs; in a snapshot graph, the only indication of time was a motion arrow. What if we want to draw a ripple on the surface of a pond, or some other two-dimensional wave? Then even a snapshot involves three variables: vertical displacement at each point along the length and width of the pond.

Imagine a fish tank partly full of water, into which you drop a pebble right next to the glass side. You might see something like the picture in Figure 125.1 (except for the arrows, of course). Along the glass there is effectively a snapshot graph, with a close connection to the full shape of the ripples.

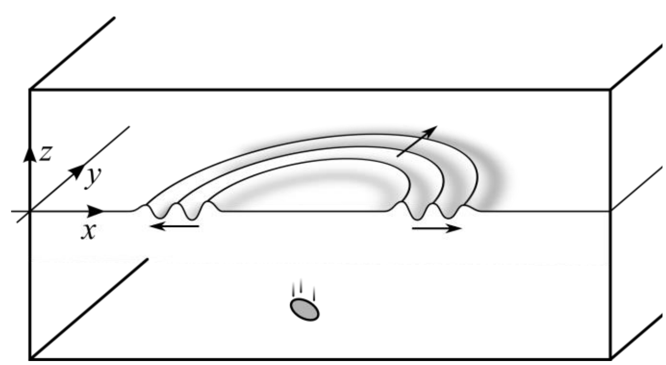

Figure 125.1

Ripple on surface of water in tank, due to a pebble dropped near the glass wall.

The axes on the left of the figure define some coordinates: *x* and *y* could be used to describe different places on the water surface, while the wave disturbance moves the water surface to different values of *z*.

The circular lines in Figure 125.1 mark the places where the *z* value of the water surface is a **local maximum.** That is, for each ring, *z* is larger than any other points very close by. Look closely where the ripple meets the glass: the peak *z* of the outer ring is a tiny bit smaller than the peak *z* of the inner ring. So, the peak *z* of the outer ring is not the largest on the whole water surface, yet it is still a *local* maximum. These rings are called **wave crests** or **wave fronts**. If we think of the ripples on the glass as being like cosine curves, then another definition of a wave crest is a connected set of points where the reduced phase is 0°.

[38] For the purist: Starting with sinusoidal displacement, this claim of sinusoidal density change directly contradicts the earlier observation that density graphs emphasize compression over rarefaction. However, sound waves have very small amplitudes, with the result that the displacement and density graphs are both very close to sinusoidal.

Occasionally we need to refer to the **local minima** that occur between the wave crests. Those are called the **wave troughs.** In relation to the cosine function, a wave trough is a connected set of points where the reduced phase is 180°.

Figure 125.2 shows the same situation from directly above, with two strategies that avoid making a perspective drawing. Part (a) shows the wave using a **grayscale,** where the height of the water at each point is represented by a color ranging from white (highest) to black (lowest), with the equilibrium height being medium gray. This method does allow for showing the full variation of the disturbance from place to place, but it would be very hard to draw with a pencil.

In Figure 125.2(b) the wave is not shown realistically, but it is instead schematically represented by its wave crests. By showing only wave crests, these diagrams have essentially given up on showing any detail of how the disturbance varies with position. Since the *z*-axis now points straight out of the page, it is drawn as if all you can see is the point of its arrowhead: a dot with a circle around it. As with snapshot graphs of one-dimensional waves, the direction of wave motion is shown by an arrow, although for 2D waves multiple arrows in different places can be helpful.

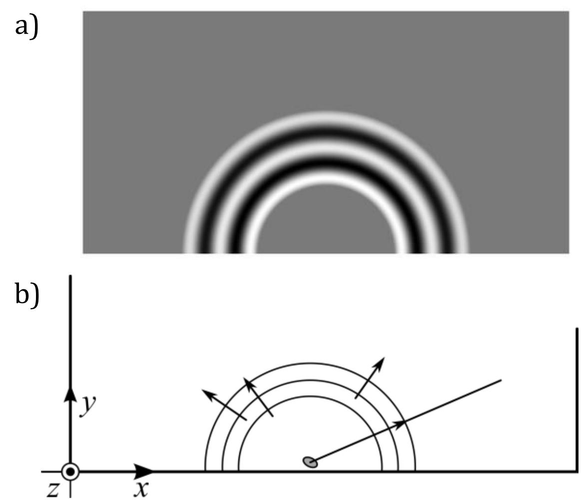

Figure 125.2
Same water ripple as in Figure 125.1, shown from directly above and more schematically. (a) In grayscale. (b) Wave crests only.

The line on the right of Figure 125.2(b) is not quite the same as the arrows. This is a **ray,** a line with an associated direction which traces the motion of the wave over a long time interval, from where it started to where it will be later. All the rays for a wave start at the wave's source. In this case the ray is straight. As described in Chapter 8 this is not always true, but even when rays are bent, it is always true that the rays are perpendicular to the wave crests. That is, wherever a ray crosses a crest, the cross makes four 90° angles, even if the line and/or the crest are curved.

Since rays can't be seen in a real physical wave, the "rays perpendicular to crests" rule might seem arbitrary. The significance is clearer for waves with boundaries. For example, imagine that Figure 125.3 is an attempt to show a beam of light shining through a window. The top and bottom rays define the edges of the beam, but the wave crests have been drawn with an impossible angle. A beam with well-defined edges is harder to achieve with sound, for reasons detailed in Chapter 164. A theoretical foundation for rays and crests being perpendicular is described in Chapter 157.

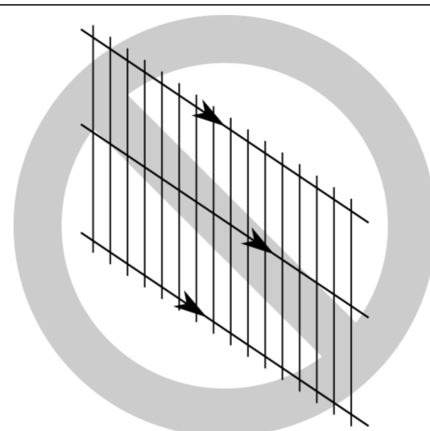

Figure 125.3
An impossible arrangement of wave crests and rays.

Figure 125.1 and Figure 125.2 are drawn as if the pebble were a periodic source, with the period being the time between creating one crest and the next. The duration is two-and-one-half periods. That's not very realistic for a dropped pebble, but it provides an example for the description of periodic sources. In this case, the space between two adjacent wave crests contains one cycle of the wave. The distance from one wave crest to the next, measured along a ray, is therefore the wavelength of the wave. In between the wave crests, the wave is causing a full cycle of the disturbance that makes the wave.

The pebble is an example of a **point** source**,** first described in Chapter 8. The resulting wave is called a **circular wave,** after the shape of the wave crests. Section 53b qualitatively describes how, as each crest expands out, its energy is spread thinner, although in this 2D case the math is somewhat different. This is why, even though we are pretending that the pebble is a periodic source, the outer ring in Figure 125.2 has a slightly smaller amplitude than the inner ring. A periodic point source produces a wave that is technically not periodic. But it's as close as one can get while still obeying energy conservation.

125b. Drawing Constrained 3D Waves

Representing waves in a three-dimensional medium is even harder than in two dimensions. A full treatment will be left to Chapter 151. But sometimes we want to represent waves that, while technically in a three-dimensional medium, are limited in a way that makes them effectively 1D or 2D. For example, air in a tube is a 3D medium. But if all variations in the medium happen over distances that are much longer than the width of the tube, then such a wave is effectively one dimensional. In those cases, we can represent the waves using methods from 1D or 2D.

Compression waves (longitudinal waves) are not so common in one or two dimensions, but in three dimensions they're much more common. A grayscale representation, as in Figure 125.2(a), is especially attractive for compression waves. Representing displacement through color is abstract; but representing high density with darkness feels much more natural. Remember, though, that the relationship between disturbance and color is arbitrary. It could be that lighter color represents higher density, and darker represents lower density.

Chapter 124 shows how the disturbance in these waves, can be described by either displacement or density of the medium. Which should be chosen for these pictures? For example, does the term "wave crest" refer to maximum compression or maximum displacement? For many purposes, it actually doesn't matter. Just knowing that the crests represent points with the same reduced phase, spaced by one wavelength, is enough to answer many questions. But in cases where the distinction is important, the picture needs to be specific. Density is more commonly represented than displacement, but neither is guaranteed.

Chapter 126. Pressure

126a. Pressure Basics

Usually, people think of air as something that flows and blows. But put some air in a balloon, and it is definitely springy.

When a balloon is squashed by a hand, the air inside pushes the hand back towards an equilibrium position. But the way that **fluids** (the category including both liquids and gasses) push is somewhat different from the way a spring pushes. The tip of a finger can be pushed much further into a balloon than the palm of a hand can be, even if they are pushing with the same force. This is because the push of the air is spread over the surface area of the balloon. The force with which the air is pushing back on the finger or palm is proportional to the contact area involved. (At least, this is true before the balloon has been deformed very much. At some point the stretching rubber of the balloon itself also starts to play a significant role.)

Once again, when two quantities are proportional to each other, we find it useful to take their ratio. When a fluid or surface is providing a force F that is distributed over an area A, their ratio is defined as the **pressure** of the fluid or the pressure applied by the surface,

$$p = \frac{F}{A} \quad . \tag{126.1}$$

This equation defines the SI derived unit of pressure to be a newton per square meter. This unit is given the root name pascal, abbreviated Pa. One pascal is a very small pressure. If you press your thumb very hard on a table, you can probably apply a pressure of 1 MPa!

The air in our balloon is pushing outward with nearly the same pressure in all directions. The rubber of the balloon is of course applying a pressure inward, so that things are in a steady equilibrium. If you push on the balloon with either your finger or your palm, the air applies the same pressure to either one (transmitted through the rubber). However, because the fingertip has a much smaller area, it experiences a much smaller *force* than the palm does, and it is therefore able to push into the balloon further.

While the balloon is being squashed, the air also pushes in other directions besides against the hand, which is why the balloon spreads out when squashed. It is a special property of fluids (including both liquids and gasses) that an increase in pressure in one direction and location always implies the same increase in pressure in all directions and everywhere in the fluid. In fact, some would say that is the defining property of a fluid. This is **Pascal's Law.**

The balloon's rubber is not the only thing applying pressure inwards; the air outside the balloon is doing so as well. The atmosphere is always compressing everything, including us, with a pressure of roughly $100000\text{ Pa} = 100\text{ kPa}$. This is the barometric pressure that meteorologists speak of, and it varies quite a bit with the weather, usually staying between 95 kPa and 105 kPa. (In the United States, different units are typically used, so these numbers may not sound familiar to you.) This is quite a large pressure. For instance, on the pad of your thumb, the air is applying a force of roughly

$$F = pA = (100\text{ kPa})(10^{-2}\text{m})^2 = 10\text{ N} \quad , \tag{126.2}$$

or about 2.5 pounds. How are you able to resist this much force? Because an equal force is being applied to your thumbnail! Because the air is all around us, pushing from both sides, we almost never notice the pressure. In fact, we are more likely to notice the *absence* of the pressure. Sticking your hand over the nozzle of a typical vacuum cleaner will reduce the pressure on that patch of skin to about one-third of atmospheric pressure.

Because of this pre-existing (or ambient) pressure all around us, we are usually more interested in changes of pressure Δp. If you push on a balloon, applying a healthy 100 N with the palm of your hand (sized $10\text{ cm} \times 10\text{ cm}$), you are increasing the pressure by

$$\Delta p = \frac{F}{A} = \frac{100\text{ N}}{(0.1\text{ m})^2} = 10\text{ kPa} \quad , \tag{126.3}$$

from about 100 kPa (the air that was pressing there before you applied your hand) to about 110 kPa. If you cover the vacuum cleaner nozzle with your hand, the pressure on your hand decreases, so that

$$\Delta p = 33\text{ kPa} - 100\text{ kPa} = -67\text{ kPa} \quad . \tag{126.4}$$

This deviation of pressure from ambient is called the **gauge pressure.** That's because this is the measurement made by a pressure gauge, like you might use on a car tire. When it is necessary to distinguish the two, the total pressure is called the **absolute pressure.** "Ambient" here is usually taken to include any effects from weather or altitude. That is, as a weather system passes over your house, the absolute pressure of the air changes, but the gauge pressure of the air remains zero.

126b. Extra: Pressure and Depth

If you fill a balloon with water, it will tend to bulge out at the bottom. This is because the pressure in the water is higher near the bottom than at the top. For the purposes of this book, it is not necessary to have equations for this effect. We just need to know that the pressure in a fluid increases as one moves down in the fluid. The rate at which this happens is proportional to the fluid density, so it is much more noticeable for liquids than for gasses.

Many people are quite familiar with this from swimming. You do not have to dive very deep before the increased pressure hurts your ears. A depth change of even 10 cm can cause a pressure change that is significant for the topic of sound. For gasses, the pressure changes much more slowly with height. In air near the surface of the earth, it is often a sufficiently accurate approximation to say that the pressure is the same everywhere in the air. Changes in altitude of 100 m or so are needed to observe a 1% change the atmospheric pressure.

Pascal's Law can be a little tricky, because it does ***not*** say that the pressure in a fluid is the same at all points and in all directions. After all, the pressure in the water balloon is larger at the bottom. Strictly, Pascal's Law only says that pressure *changes* are the same everywhere in a fluid.

However, if we are ignoring the pressure changes with height, then that model, *along with* Pascal's Law, does lead to the conclusion that the pressure is the same at all points and in all directions. This would be a good model for an air-filled balloon.

Chapter 127. Density and Pressure

If you push a balloon down on a tabletop, then it gets shorter, but it also gets wider. This makes it difficult to be sure how the balloon's volume is changing. Instead, suppose that the same balloon is put inside a box of the same size as the balloon and with an open top. This will make it much harder to compress the balloon (people are often surprised at how much harder). But it still can be compressed, and now the increased pressure clearly reduces the balloon volume. Eq. 117.1 then implies that increased pressure is associated with increased density.

Unfortunately, it turns out that the density and the pressure in a medium are *not* proportional to each other. Section 132b explores this a bit more deeply. But for studying acoustics, what really saves the day is that only small changes from equilibrium are of interest. A small gauge pressure Δp will result in a small density change $\Delta\rho$. As long as those are small compared to the equilibrium values, then it is a very good model to say that the changes are proportional

$$\Delta\rho \propto \Delta p \quad . \qquad (127.1)$$

A compression wave in a fluid, then, can be considered as either a wave of density or a wave of pressure, on top of being a longitudinal displacement wave. For example, in Figure 124.2(d) the sound wave is graphed in terms of linear density change. But that's exactly the situation for which Eq. 118.3 tells us that linear and volumetric density are proportional, so that snapshot graph could just as well be a graph of the volumetric density change. And a graph of the gauge pressure in the sound wave would also look just like Figure 124.2(d). Any representation of a compression wave could equally well be understood to be showing either density or pressure.

The existence of sound waves in fluids shows that there must be a restoring force that pushes the fluid towards an equilibrium state. An advantage of the pressure perspective is that it makes that restoring force more explicit. Imagine two small neighboring chunks of air, and suppose that a disturbance makes one expand at the expense of the other contracting. This would cause the pressure in them to decrease and increase, respectively, which would push the boundary between them back towards its original position. Thus, it's not so much the pressure, but rather the pressure difference that provides the restoring force.

Chapter 128. Pressure and Velocity Microphones

Chapter 102 noted that the acoustomechanical transducer in an audio device must have a significant surface area that interacts with the air. More commonly, a diaphragm interacts with the air through pressure. Either the diaphragm pushes and pulls on the air (for a speaker), or the air pushes and pulls on the diaphragm (which makes it a **pressure microphone**). The fact that these forces relate to air pressure through Eq. 126.1 is one indication of why a diaphragm's area is an important characteristic.

But for the acoustomechanical transducer in a microphone, there is another alternative. The transducer of the ribbon microphone in Chapter 107 is in the category of **velocity microphones** (also called **pressure-**

gradient microphones), which are better described as responding to the movement of the air, rather than the pressure.

The distinction is perhaps clearest in the situation depicted in Figure 128.1, where a sound wave is passing across the front of two microphones. In the sound wave, the darker grays represent compression ($\Delta p > 0$) and the lighter grays represent rarefaction ($\Delta p < 0$). As the sound passes the crystal microphone in the middle, the compressed air pushes on the diaphragm; one half period later, a rarefaction would be pulling on the diaphragm. The microphone will therefore pick up the sound.

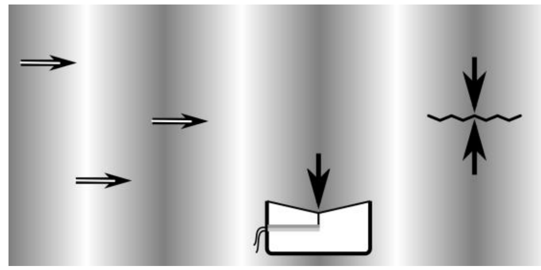

Figure 128.1

Sound wave passing left-to-right past crystal microphone and ribbon microphone, both facing upwards. Solid arrows indicate air pressure pushing on microphone elements.

But as the sound passes the ribbon microphone on the right, the compressed air pushes on both sides, so that there is no net force on the ribbon. A similar cancelation occurs when the rarefactions arrive. This microphone will not pick up any sound. In order to respond to the sound, the ribbon must be turned 90°, so that the ribbon can move back and forth with the longitudinal displacements of the air.

Clearly, how well this microphone responds to a sound depends on its orientation. Actually, that's true for both diaphragm and ribbon microphones, and is described by the microphone's **pickup pattern.** But that topic isn't included in this book. This example is intended only to highlight the difference between pressure microphones and velocity microphones.

Chapter 129. Intensity and Pressure

All compression waves, including sound, are longitudinal waves as well. But focusing on the pressure variations in sound, as opposed to other ways of measuring sound's disturbance of the air, offers some distinct advantages. For example, most sensors for detecting sound (including the ear drum) do so by responding to the forces that come from changes in pressure.

Another advantage involves calculating sound intensity from sound amplitude. Proportion 32.1 introduced a general relationship, which remains true here: sound intensity (as an energy-related quantity) is proportional to the square of any type of amplitude. But complications can arise, because energy-related quantities can also depend on the frequency.

Conveniently, sound intensity can be calculated from the pressure amplitude *alone*, without the frequency. The equation only requires a few constants describing the medium:

$$I = \frac{p_m^2}{\rho_0 s} \quad , \tag{129.1}$$

where ρ_0 is the equilibrium density of the medium, s is the speed of sound in the medium (which we assume is non-dispersive), and p_m is the **pressure amplitude.** Pressure amplitude is the maximum change in pressure (away from average) in the wave. That is, p_m is the maximum value of the gauge pressure Δp. For air under normal terrestrial conditions, the denominator in Eq. 129.1 has roughly the value

$$\rho_0 s \approx 400 \, \frac{\text{kg}}{\text{m}^2\text{s}} \quad , \tag{129.2}$$

although the precise value depends on variations in temperature and atmospheric pressure.

A sound with a pressure amplitude of $p_m = 1$ Pa, a tiny variation in the ambient air pressure, would be a very loud sound. To practice using these equations, you could determine the range of pressure amplitude that encompasses the limits of human hearing described in Chapter 51.

Chapter 130. Intensity & Displacement

To explicitly show how convenient Eq. 129.1 is, the equation relating sound intensity and displacement amplitude x_m is

$$I = \frac{\rho_0 s}{8\pi^2} f^2 x_m^2 \quad . \tag{130.1}$$

The dependence on frequency f is mixed in with the dependence on amplitude.

Although it is more complicated than Eq. 129.1, Eq. 130.1 does help to understand one feature of sound system speakers that you may have noticed. Speakers that are good at producing low frequency, bass pitches are generally large. There are several reasons behind this fact. One is that the large speaker size makes large amplitude vibrations easier, which in turn allow the speakers to produce sound waves with larger displacement amplitudes. Equation 130.1 reflects that in order to have the same intensity, lower frequency sounds require these larger displacement amplitudes.

Chapter 131. Pressure Spectra

131a. Pressure and Intensity Spectra

Chapter 42 shows how the idea of a spectrum can be used to understand a complex oscillation, breaking it into a collection of sinusoidal parts, each with a specific frequency and size. Different ways to represent the size are explored in Chapters 42 and 61. They fall into two categories, energy-type and amplitude-type. Although proportion 32.1 gave a generally applicable relationship between those categories, it's sometimes difficult to use due to a simultaneous relationship between energy and frequency.

But because Eq. 129.1 does not involve frequency, there is a straightforward relationship between the intensity spectrum of a sound and its pressure amplitude spectrum. Figure 131.1 shows an example. Notice that the heights of the peaks keep the same qualitative relationships; for example, the tallest peak in the pressure spectrum remains the tallest in the intensity spectrum. But the relative sizes of the peaks are quantitatively different.

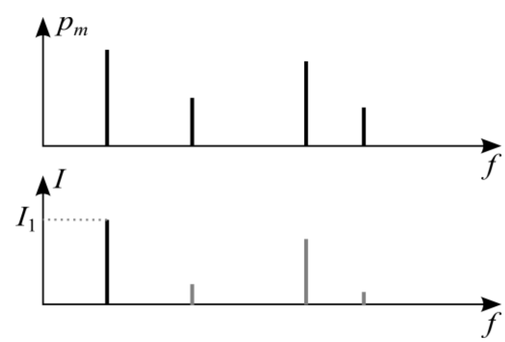

Figure 131.1

Pressure amplitude spectrum and intensity spectrum for the same sound.

If the numerical vertical scale is known for one of the graphs in Figure 131.1, then Eq. 129.1 can be used to calculate the peak heights for the other graph. But there are qualitative relationships that can be used even without vertical scales. Because the $\rho_0 s$ in Eq. 129.1 is a constant, the other variables are proportional to each other,

$$I \propto p_m^2 \quad , \tag{131.1}$$

which is just a specific example of proportion 32.1. On the most qualitative level, this tells us that distinctions between peak heights are more emphasized on intensity spectra. The proportion implies the ratio equation

$$\frac{I_n}{I_1} = \left(\frac{p_{mn}}{p_{m1}}\right)^2 \quad . \tag{131.2}$$

For example, since the second partial in Figure 131.1 has half the pressure amplitude of the fundamental, that partial has an intensity that is $(1/2)^2 = 1/4$ of the fundamental's intensity.

131b. Extra: Incomplete Spectrum Information

When measuring sound spectra, there are often unknown scaling factors (such as the gain of a microphone, see Section 88b), with the result that only the relative peak sizes are known. This is equivalent to not knowing the scale on the vertical axis. In such situations, the proportion Eq. 131.2 can be more useful than the equation Eq. 129.1.

Suppose that we know the shape of a pressure amplitude spectrum, and we also know numerically the intensity of the fundamental I_1. That is, the unknowns in Figure 131.1 are the pressure scale and the gray intensity peaks. Since the pressure amplitude *ratios* can be known without knowing the pressure amplitude scale, Eq. 131.2 can be used to calculate the intensities of the other partials.

But it would be strange to know the intensity of just one partial. It's much more likely to know the total intensity I_{tot}, which for example could be obtained with a sound level meter. How can this be used along with a scale-less pressure amplitude spectrum to calculate intensities of the partials?

That calculation could be approached in the following way, using the idea from Chapter 61 that, unlike amplitude partials, the intensities of the partials can be summed to get the total intensity. First, we guess a value for the intensity of the fundamental I_1^g. We know that this is different from the real fundamental intensity by an unknown multiplier K. But the guess can be used with Eq. 131.2 to calculate intensities for the other partials I_n^g. These are all wrong by the same factor, as described by

$$I_n = K I_n^g \quad . \tag{131.3}$$

This means that the total guessed intensity is also wrong by the same factor, giving the equations

$$I_{\text{tot}} = I_1 + I_2 + \cdots = K I_1^g + K I_2^g + \cdots = K I_{\text{tot}}^g \quad , \tag{131.4}$$

$$K = \frac{I_{\text{tot}}}{I_{\text{tot}}^g} \tag{131.5}$$

Since the true total intensity is known, Eq. 131.5 can be used to find the correction factor K, and Eq. 131.3 can then be used to go back and correct the guess-based intensities of the partials.

Chapter 132. Squeezing Fluids

132a. Changing Fluid Volume

Squeezing a balloon is similar to squeezing a spring. If the balloon is just on a surface, then it will spread out when the top is pushed down. Putting the balloon in a box with the same size and with an open top, to prevent the spreading, makes it resist the pushing more. The same force will then result in a smaller displacement of the top of the balloon. Making an analogy with a compression spring, putting the balloon in the box results in a higher spring stiffness constant.

The crucial difference between those situations is freedom in the width. Spreading sideways is a way that the balloon tries to keep its **volume** the same. In fact, if you apply the same force with your hand in both situations, the resulting volume of the balloon will be the same, even though the displacement of the balloon's top is larger without the box.

A better quantity to follow is the *change* in the volume ΔV. This plays a role similar to the displacement of a spring's end Δx. But by focusing on change in volume instead of displacement, we can simultaneously discuss the situations with and without the box, which otherwise would have to be dealt with separately. This type of unification is a general goal in the science of physics. By finding a viewpoint that encompasses the largest number of situations, we find equations and concepts that are more powerful.

We can widen our viewpoint even further. Until now, we have only considered a balloon of a single size. Call that balloon *A*. Now consider a number of other balloons with a variety of sizes. In particular, suppose that balloon *B* has twice the volume of balloon *A*. If we now apply the same force with our hand to these two balloons, we will find that the squashed volume of *B*, and therefore the volume change for *B*, are also twice those for balloon *A*,

$$\Delta V_B = V_{Bf} - V_{B0} = 2V_{Af} - 2V_{A0} = 2\left(V_{Af} - V_{A0}\right) = 2\Delta V_A \quad , \qquad (132.1)$$

where the subscripts f and 0 are for the final and initial situations. In fact, applying the same force to many different balloons will show that the change in volume is proportional to the initial volume. As usual with proportional quantities, we can unify many different cases by focusing on their ratio.

> The most generally useful way to characterize the degree of compression of a fluid is with the **fractional change in volume** $\Delta V/V_0$, where ΔV is the change in the volume and V_0 is the uncompressed volume.

Because the units in the top and bottom of that fraction are the same, the fractional volume change is a unitless quantity. If you want to get fancy, it is known as the **volumetric strain.** But either way, there is no commonly accepted algebraic symbol for this concept.

132b. Fluid Restoring Force

When the pressure on a fluid (either gas or liquid) changes, the volume of the fluid also changes. If the changes are a small fraction of the original pressure and volume, then the volume change is *proportional to* the pressure change, as expressed by

$$\Delta p \propto -\Delta V \quad , \qquad (132.2)$$

where the negative sign has been included as a reminder that increasing the pressure causes a decrease in volume. As suggested by the preceding section, this proportion applies more broadly to fractional volume change,

$$\Delta p \propto \frac{-\Delta V}{V_0} \quad . \qquad (132.3)$$

Think of this as parallel to Hooke's observation that, for a spring, force applied is proportional to a change in length. As with our development of Hooke's Law, we can measure the ratio of pressure change to fractional volume change. It's simplest to define this ratio with an explicit negative, with the equations

$$\frac{\Delta p}{\Delta V/V_0} = -B \quad \Rightarrow \quad \Delta p = -B\left(\frac{\Delta V}{V_0}\right) \quad . \qquad (132.4)$$

The quantity *B* is called the **bulk modulus** of the fluid. Much like a spring constant, the bulk modulus expresses how difficult it is to compress the fluid. We can see from the equation that since the fractional volume change is unitless, the bulk modulus has the same unit as pressure. In other references, you might find the bulk modulus represented by the letter K.

The model expressed by Eq. 132.4 is a good start, but it is subject to a few caveats. One is that *B* depends on how quickly the compression is done, compared to the rate at which the fluid can cool off when it is hotter than its surroundings. Why this should be true is well beyond the scope of this book. Suffice it to say that any changes of interest in this book are considered "fast." In other references, you might find these fast changes described by the adjective **adiabatic.**

For gasses, another catch is that the bulk modulus is not actually a constant, but instead varies with the pressure. As a result, Eq. 132.4 can only be used for gasses if the pressure stays near a particular value. The case of most interest in this book is air near 100 kPa, for which the bulk modulus for fast changes is $B_{\text{air}} = 140$ kPa. Even so, one must be on guard for applications where the pressure dependence of B cannot be ignored.

For liquids, both of these caveats apply, but they are of much less concern. Liquids are much, much less compressible than gasses, so much so that in normal situations we do not consider liquids to be "springy" at all. For instance, the bulk modulus of water is about $B_{\text{water}} = 2.15 \times 10^6$ kPa, or roughly 15000 times larger than air. This means that deviations of the bulk modulus with pressure or with speed of compression, although present, are almost always negligibly small.

132c. Extra: Density and Pressure

These ideas can be pulled together to refine the relationship in Chapter 127. If a chunk of fluid has mass m and in equilibrium has volume V_0, density ρ_0, and pressure p_0, then the math combining Eqs. 117.1 and 132.4 looks like

$$\Delta\rho = \rho - \rho_0 = \frac{m}{V} - \frac{m}{V_0} = m\left(\frac{V_0 - V}{VV_0}\right) = \frac{m}{V}\left(\frac{-\Delta V}{V_0}\right) = \rho\frac{\Delta p}{B} \quad , \tag{132.5}$$

$$\Delta\rho = \frac{\rho}{B}\Delta p \quad . \tag{132.6}$$

At first, this result looks like an equation version of proportion 127.1. But it isn't really. In order for pattern Eq. 4.26 to apply, the multiplier must be a constant, and we know that the density ρ and bulk modulus B are not constant. Even worse, to treat this as a proportion would simultaneously require that the density *is* changing ($\Delta\rho \neq 0$) and that the density *is not* changing (ρ constant).

As elsewhere in this Part, the best that can be done at the level of this book is to restrict the changes from equilibrium to be small. Then, the density and bulk modulus in Eq. 132.6 can be approximated by their equilibrium values. Luckily, that is a very good model for acoustic waves.

Chapter 133. Sound Speed in a Fluid

The sound definition in Chapter 3 states that sound waves are "longitudinal pressure waves." Such waves turn out to be non-dispersive in most fluids. Even without knowing a great deal about pressure in fluids, the basic pattern in Chapter 116, along with the ideas from Chapters 117 and 129, allow a good guess at an equation for sound wave speeds.

If neighboring chunks of fluid are out of equilibrium with each other, the higher pressure in the more compressed chunk presses on the less compressed chunk. The higher pressure one expands, the lower pressure one contracts, and they change towards equilibrium. Much like a spring constant, the quantity that describes the overall strength of the restoring force is the bulk modulus B.

Since sound travels through three dimensions, the massiveness of the fluid is appropriately described by the volumetric mass density ρ. Following the model in Eq. 116.1, we can guess that the speed of a sound wave in a fluid might be

$$s_{\text{fluid}} = \sqrt{\frac{B}{\rho}} \quad . \tag{133.1}$$

Dimensional analysis can check whether this equation is sensible, by making the "calculation"

$$\sqrt{\frac{\text{Pa}}{\text{kg/m}^3}} = \sqrt{\frac{\text{N}}{\text{m}^2} \cdot \frac{\text{m}^3}{\text{kg}}} = \sqrt{\left(\text{kg}\frac{\text{m}}{\text{s}^2}\right) \cdot \frac{\text{m}}{\text{kg}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{m}}{\text{s}} \quad . \tag{133.2}$$

In this case, not only is dimensional analysis satisfied, but Eq. 133.1 turns out to be exactly correct.

Just because Eq. 133.1 is correct, it's not necessarily the most useful equation available. For liquids it is indeed quite useful. Even though pressure in a liquid increases significantly with depth, the properties in Eq. 133.1 do not change significantly.

On the other hand, for gasses we face a difficulty. For a gas, the quantities in Eq. 133.1 do vary with pressure, as warned in Section 132b. In some situations such changes are small enough to ignore. But because two such properties both appear in Eq. 133.1, their effects get entangled. Luckily there is a better option. It turns out that the speed of sound in a gas does not depend on the gas pressure at all. The result is described in Chapter 120.

Chapter 134. The Invisible Spring

To clarify the connection between bulk modulus *B* and spring stiffness constant *k*, consider the example illustrated in Figure 134.1. Instead of squashing a balloon, we'll imagine an airtight rectangular box in order to keep the equations simple. The box is *w* wide (side to side) and *l* long (front to back). The lid of the box can move up and down like a piston, without letting any air through the joint between lid and box. While undisturbed, the lid rests at a height y_0 above the bottom of the box, supported by the air inside.

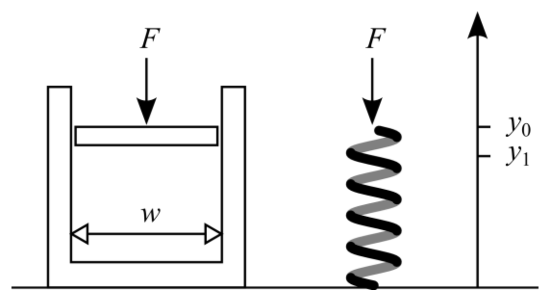

Figure 134.1
Compression of fluid compared to compression of a spring.

If a force *F* is applied to the lid of the box, then the lid will move down a bit to a height y_1. If we knew all the other quantities, how could we find y_1? First, we need to consider the *change* in the volume,

$$\Delta V = V_f - V_i = wly_1 - wly_0 = wl\,\Delta y \quad . \tag{134.1}$$

(Note that for the specific scenario described, both ΔV and Δy are negative.) Now Eq. 132.4 can relate the volume change to the additional pressure, and Eq. 126.1 will relate the pressure to the applied force. Putting all these together,

$$\frac{F}{wl} = \frac{F}{A} = \Delta p = -B\left(\frac{\Delta V}{V_i}\right) = -B\left(\frac{wl\,\Delta y}{wl\,y_0}\right) = -B\frac{\Delta y}{y_0} \quad , \tag{134.2}$$

which uses the fact that the area of the box lid is $A = wl$.

The behavior of this box is very similar to that of a compression spring. We can even rearrange Eq. 134.2 so that it looks like Hooke's Law,

$$F = -\left(wl\frac{B}{y_0}\right)\Delta y \quad , \tag{134.3}$$

demonstrating that the box behaves exactly like a spring, with a spring constant

$$k = wl\frac{B}{y_0} \quad . \tag{134.4}$$

Don't go memorizing that equation, as it is very specific to this rather artificial situation. The important point is the connection between springs and gases.

Chapter 135. Helmholtz Resonator, Advanced

135a. Derivation

In Chapter 28 the Helmholtz resonator was introduced as a device with a preferred frequency for air vibrations. At this point in the book, we are now prepared to understand in more detail the formula for that preferred frequency, Eq. 28.1.

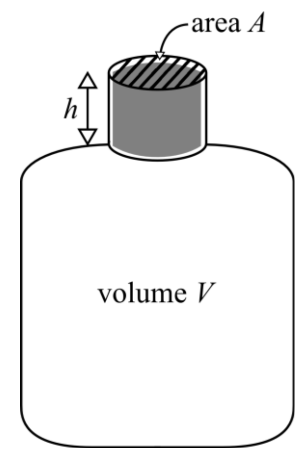

Figure 135.1
Dimensions of a bottle, considered as a Helmholtz resonator.

Figure 135.1 duplicates Figure 28.1, showing the elements of the resonator. The basic idea is that the air in the neck of the resonator (the gray cylinder in the figure) forms a mass that oscillates up and down. The restoring force is supplied by the air in the body of the resonator, which attempts to maintain a constant volume.

To describe the restoring force, we proceed much as in Chapter 134. When the neck air (gray cylinder) is in the equilibrium position, there is no net force on it because the body air pushes up with the same size force as the external air pushing down. Suppose the neck air were to move upwards a small distance Δy. At the bottom of the cylinder, this would add a volume

$$\Delta V = A\,\Delta y \tag{135.1}$$

to the body air. This reduces the pressure in the body (Δp as given by Eq. 132.4), so that the unchanged external pressure provides an unbalanced force F downwards. Bringing these relationships together, we have

$$\frac{F}{A} = \Delta p = -B\left(\frac{\Delta V}{V}\right) = -B\left(\frac{A\,\Delta y}{V}\right) \quad , \tag{135.2}$$

$$F = -\left(\frac{A^2 B}{V}\right)\Delta y \quad , \tag{135.3}$$

where the force is negative $F < 0$ because it is downwards. This shows that there is a Hooke's Law restoring force on the neck air with a spring stiffness constant

$$k = \frac{A^2 B}{V} \quad . \tag{135.4}$$

It might seem strange that this is proportional to the square of the neck area. A larger area provides more resistance both because any pressure change effects the whole area, and because it changes the internal volume by a larger amount.

The mass of the neck air is simply related to its density with the equation

$$m = \rho\,Ah \quad . \tag{135.5}$$

From these, we can use Eq. 27.2 and Eq. 133.1 to find the natural oscillation frequency to be

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{A^2 B/V}{\rho\,Ah}} \quad , \tag{135.6}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{B}{\rho}\frac{A}{Vh}} = \frac{1}{2\pi}s\sqrt{\frac{A}{Vh}} \quad . \tag{135.7}$$

Eq. 135.6 keeps the restoring force part and mass part of the equation separate. For instance, notice that it is *not* correct to claim that the "massiveness" of the system comes from the volume V, even though it appears in the denominator of Eq. 135.7. However, the various variables do simplify quite a bit in Eq. 135.7, which has used Eq. 133.1 to introduce the speed of sound to the equation. The sound speed doesn't appear because of anything actually moving at that speed in Figure 135.1; it is more a mathematical coincidence, arising from the fact that the same type of restoring force and massiveness are involved in both the Helmholtz resonator and in traveling sound waves.

135b. Extra: Limitations

For our model of the Helmholtz resonator to work, the wavelength associated with its frequency has to be much longer than the longest dimension of the body volume. If the wavelength were shorter than that, then we would have to consider waves flying around inside the body, bouncing off the walls. Suppose that the body is fairly compact, for instance a cube with side length x_V, so that $V = x_V^3$. Then this requirement can be combined with Eqs. 121.2 and 135.7 to give

$$x_V \ll \lambda = s/f = 2\pi\sqrt{\frac{Vh}{A}} \quad , \tag{135.8}$$

$$x_V^2 \ll 4\pi^2 \frac{x_V^3 h}{A} \quad , \tag{135.9}$$

$$x_V \gg \frac{A}{40\, h} \quad , \tag{135.10}$$

using the fact that $\pi^2 \approx 10$. Broadly speaking, the dimensions of the body volume must be significantly larger than the dimensions of the neck. Even though a less compact body shape makes the requirement more stringent, pretty much any shape with a separately identifiable neck (or hole) and body will satisfy this inequality.

Chapter 136. Deforming a Solid

136a. Compressing Solids

Sounds certainly pass through solids just as well as through liquids and gasses, but describing sound in solids is much more complicated. The very defining characteristic of solids, rigidity, means that they resist deformation in multiple ways. For example, a stick can be bent, and spring back to straight — you can't do that with water! Resisting deformation means there is a restoring force, one of the fundamental requirements for waves. Different types of restoring forces mean that there are different types of waves in solids.

For sound, the most important deformation of a solid medium is compression, since that matches what happens in fluids. Consider pushing down on the top of a wooden block. The block will compress just a tiny bit, while providing a restoring force back up on your hand. The best way to describe the relationship between the deformation and restoring force is sort of part way between the case of a spring and the case of fluids.

- We might need to compare different blocks, with different sized tops. Therefore, as with fluids, it is most generally useful to focus on the pressure being applied (that is, the force applied divided by area). Since the blocks retain their shape, it is easier than for fluids to identify which area to use:

instead of being concerned with how the force is applied, the appropriate area is simply the area of the top of the block.

- As a block's height is reduced, its width increases, somewhat like a squashed water balloon. However, while the water seeks to maintain the same volume (and it would succeed if it weren't for the rubber of the balloon pressing in), the rigidity of the block works against the width increase. Even without the help of external walls, the block volume decreases.
  In the end, scientists and engineers have found it most convenient to measure the compression by the change in height of the block Δx, just as with a spring. A benefit of this is that it avoids the complications of how the volume is changing.
- Somewhat parallel to the volume in the fluid case, the change in height of the block is proportional to the initial height of the block. Under the same pressure, a taller block with a larger initial height x_0 will compress by a larger amount, while a shorter block with compress by a smaller amount. Thus, it is handy to work with their ratio, $\Delta x/x_0$, which is called the **strain.** The strain is traditionally represented by the Greek letter epsilon, ε, although in this book we won't need that symbol.

Put these ideas all together, and the pressure and compression are related by the equation

$$\Delta p = -E\frac{\Delta x}{x_0} \quad , \qquad (136.1)$$

where E is called **Young's modulus**, which has the unit pascal, just the same as pressure and bulk modulus. Each substance has a characteristic value for Young's modulus, just as each substance has a density. Typical values of Young's modulus are a little larger than typical bulk moduli for liquids. For example, Young's modulus for wood is $E \approx 13 \times 10^6$ kPa and for steel is $E \approx 200 \times 10^6$ kPa.

The traditional choice of E as the variable for Young's modulus seems a little unfortunate, since it conflicts with energy. The variable Y is available with extremely few conflicts, and some references do use it. But sometimes we just have to work with what history hands us, so this book will use E.

136b. Squeezing Solids

Instead of compressing a solid block in one direction, we could press in on all sides of it. Chapter 129 describes changing the volume of a fluid a few different ways. The details didn't matter, because a fluid does not try to keep the same shape, and Pascal's Law tells us that the pressure increase where it is pressed is communicated throughout the fluid. But a solid block doesn't react like that. In order to do this experiment, we would need to actively press on all sides of the block. Probably the most practical way to do this is to submerge the block in a liquid. That way, when the pressure of the liquid is increased, the pressure with which the liquid is squeezing the block increases uniformly on all sides.

Other than having to be careful about how the pressure is applied, this experiment is directly analogous to the one in Section 132b. A uniform increase of pressure results in a volume decrease, and the **bulk modulus** B can be defined exactly as in Eq. 132.4. A solid material usually has a bulk modulus that is somewhat close to its Young's modulus, but either one could be larger than the other.

136c. Other Solid Deformations

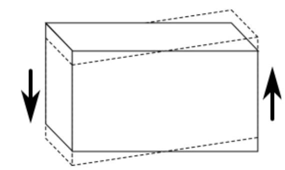

Figure 136.1
Shear deformation. Solid lines show original block, dashed lines show deformed shape of block. Arrows show external forces causing deformation.

Solids also resist being bent, twisted, and any other attempt to change their shape. There is one deformation that has particular importance for sound. This is the **shear deformation** illustrated by Figure 136.1, in which opposing sides of a block are forced parallel to the faces and in opposite directions. The resulting shape change is special because the volume of the block is unchanged, which makes this a good counterpoint to the compressions discussed in preceding sections. (To convince yourself that the volume is unchanged, start with the formula for the area of a parallelogram.)

This book will not go into the details. Suffice it to say that this type of restoring force is characterized by a value called the **shear modulus,** symbolized by G. This has the same units as the other moduli, pascals. For any given material, its shear modulus is typically around three-eighths of its Young's modulus, reflecting that it is easier to change a solid's shape than to change its volume.

Chapter 137. Sound Speed in a Solid

How does a disturbance move through a solid material? Chapter 136 told of several different ways to deform a solid, and as a result there are several different answers to that question. In some cases, the wave motion is even dispersive, with the wave speed depending on the shape of the wave as well as the type of disturbance and the properties of the medium.

A derivation of how the restoring forces in Chapter 136 cause various wave speeds is beyond the level of this book. But the basic pattern in Chapter 116, along with the ideas in Chapters 117 and 136, allow good guesses. In all of the types of disturbances, the approach described here involves the three-dimensional bulk of the material, so the massiveness is appropriately described by the volumetric mass density ρ.

The definition of sound in Chapter 3 specifically refers to "longitudinal pressure waves." As with both gasses and liquids, solid media can support such **compression waves**. Suppose that a hammer hits a solid rod squarely on the end, so that the material of the rod is compressed. A pulse as illustrated in Figure 137.1(a) might result. As described in Section 136a, in the place where the rod is compressed (horizontally in the figure) it bulges somewhat. Notice also the stretched portion, where the rebounding rod overshot equilibrium.

The resulting restoring pressure is characterized by Young's modulus E. We can guess, based on the pattern in Eq. 116.1, that the speed of this sound wave will be [39]

$$s_{\text{rod}} = \sqrt{\frac{E}{\rho}} \quad . \tag{137.1}$$

If the end of the same rod is instead vibrated sideways, perpendicular to the rod's length, then we might imagine a disturbance like that shown in Figure 137.1(b). The deformation shown there is of the shear type shown in Figure 136.1. But this is nearly impossible to actually create. Hitting the rod end with a hammer on the side will instead result in a bending deformation, somewhat like that shown in Figure 137.1(c). This bending is the sort of motion that occurs in such instruments as xylophones. But the restoring forces are more complicated than the level of this book, and the propagation of disturbances is dispersive. No single wave speed can be identified.[40]

[39] L. E. Kinsler et al., *Fundamentals of Acoustics,* 3rd ed. (New York: John Wiley and Sons, 1982), 59.
[40] Ibid., 71.

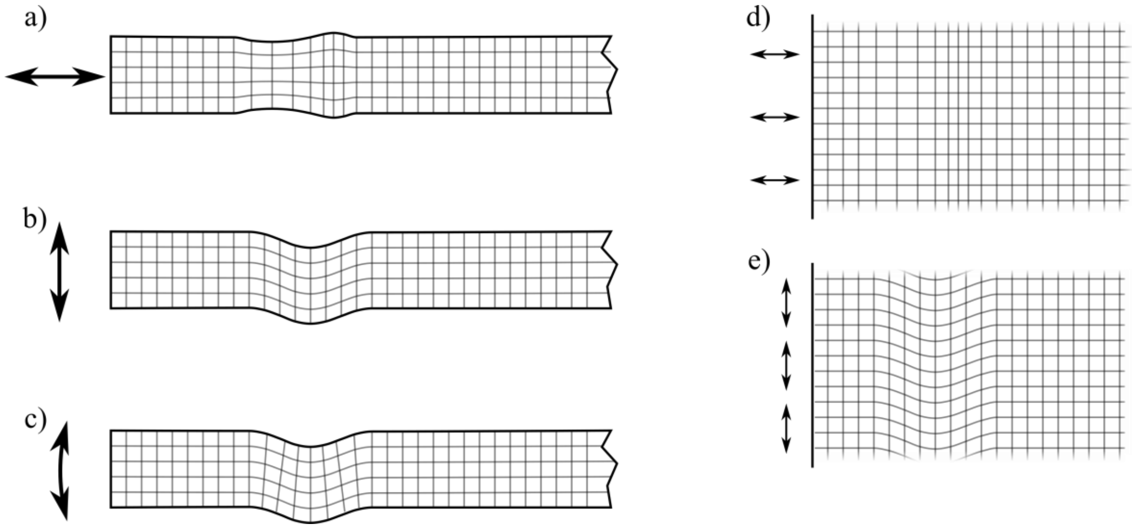

Figure 137.1

Various types of wave pulses in solid media. All displacements are unrealistically large to make them visible. (a–c) End of a solid rod. (d & e) Surface of a large solid medium. Double arrows show vibration direction, of end or surface, that might create pulses. (a & d) Compression pulse. (b & e) Shear pulse. (c) Bending pulse.

However, if the medium is much wider than the typical length of the disturbance, the bending deformation can't happen. Then it is possible to have a **shear wave** such as illustrated in Figure 137.1(e). The shear modulus from Section 136c describes the restoring force. Following the pattern in Eq. 116.1 we can guess that the speed of this **shear wave** might be

$$s_{\text{shear}} = \sqrt{\frac{G}{\rho}} \quad . \tag{137.2}$$

Whether this wave qualifies as a "sound" wave is open to interpretation. It is certainly a wave that is closely related to vibration, but it is not a compression wave.

Figure 137.1(d) shows a compression wave in a wide solid medium. This is the wave that most closely resembles sound in a gas or liquid. Because the medium is wider than the typical length of the disturbance, there is no room for the sideways bulging and narrowing that happened on the rod in Figure 137.1(a). As a result, Eq. 137.1 no longer applies. The bulk modulus B doesn't describe the situation either, because no pieces of the medium are being compressed uniformly in all directions. The correct speed formula for this sound wave is[41]

$$s_{\text{bulk}} = \sqrt{\frac{B + \frac{4}{3}G}{\rho}} \quad . \tag{137.3}$$

It turns out that this **bulk sound speed** is always higher than the bar sound speed.

Dimensional analysis can check whether any of these equations are sensible, by making the "calculation"

$$\sqrt{\frac{\text{Pa}}{\text{kg/m}^3}} = \sqrt{\frac{\text{N}}{\text{m}^2} \cdot \frac{\text{m}^3}{\text{kg}}} = \sqrt{\left(\text{kg}\frac{\text{m}}{\text{s}^2}\right) \cdot \frac{\text{m}}{\text{kg}}} = \sqrt{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{m}}{\text{s}} \quad . \tag{137.4}$$

[41] Ibid., 137.

Not only is dimensional analysis satisfied, but it turns out that the relatively simple Eq. 137.1 and Eq. 137.2 do in fact give the right wave speeds for their respective cases. But dimensional analysis could never predict or verify either the factor of $\frac{4}{3}$ or the presence of two moduli in Eq. 137.3. They can only come from a detailed analysis of how all the pieces of the solid are interacting.

Chapter 138. 1D Wave Superposition

Non-dispersive media are usually also linear, meaning that multiple waves can travel through them without affecting each other. This is excellent model for sound unless the wave amplitudes are extremely large. Having a linear medium also has implications for how the medium is disturbed by the combination of several waves in the same location.

> For a **linear medium,** while multiple waves are in the same location, the disturbance of the medium **obeys superposition:** the total disturbance at each position is equal to the sum of the disturbances that would have resulted from each of the waves separately at that position and time.

Superposition is actually a general term in physics, used whenever simply adding things together gives the correct result for their combination. See Chapter 18 for another example. In this case, we are adding curves rather than numbers.

For a one-dimensional medium, the idea of combining multiple waves that are traveling in the same direction, or **comoving,** can seem quite artificial. Suppose that a transverse wave of some particular shape is travelling along a rope; to be specific, perhaps it is the shape of the thick black line in Figure 138.1(b), moving left-to-right along the rope but keeping the same shape. For what reason would anyone claim that it is actually two waves, moving through the rope together, rather than just one wave? There are two potential answers.

- In some situations, multiple distinguishable sources may have created the wave. For example, two side-by-side speakers could both projecting forwards down a tube of air. We can consider the component waves to be what would have happened if we turn off all but one of the sources, each in turn.

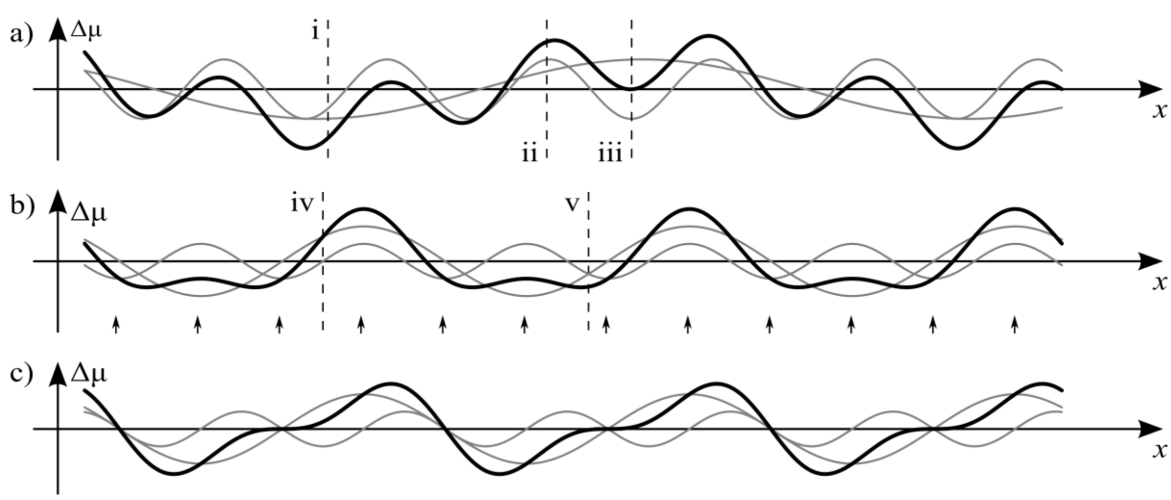

Figure 138.1

Examples of superposition of 1D density waves. Gray are contributing waves, each as it would be without the other present (their direction of travel does not matter). Thicker black is combined wave, the disturbance that results when both contributing waves are in this location.

- A more abstract reason is that this perspective may simplify predicting the effect of the wave, such as vibrating something further along. Suppose that the shape of the wave is the superposition of multiple simpler shapes. Then rather than working out the consequences of the complex shape, it may be easier to work out the consequences of these simpler shapes, and then to combine them.

The superposition of two 1D waves traveling in opposite directions, or **countermoving,** is more clearly something that might happen naturally. Such wave packets or pulses can originate in different locations, such as two ends of a rope, and then pass through one another. At any moment in time when they overlap, superposition can give the combined shape of the medium.

Figure 138.1 shows examples, using the case of compression waves. Since the examples are snapshots, it is impossible to tell whether the two contributing waves are comoving or countermoving. But this makes no difference for the purpose of finding how they combine at that moment in time. Part (a) shows the superposition of two sinusoidal compression waves with quite different wavelengths, and it is fairly easy to see how they each have each contributed to the combined wave. At each point along the position axis, the density change for the combined wave is the sum of the density changes from the contributing waves,

$$\Delta\mu_{\text{tot}} = \Delta\mu_1 + \Delta\mu_2(+\cdots) \quad , \tag{138.1}$$

where the parentheses would apply if there were more than two contributing waves. Keep in mind that the density changes, measured from the equilibrium, can be negative, which must be included when adding. At position (ii) both contributing density changes are positive, and at position (i) both are negative; in either case, adding them leads to a density change of larger magnitude. At position (iii), on the other hand, the contributions are of opposite sign, so that the sum is nearly zero.

Figure 138.1(b) shows the superposition of two sinusoidal waves of more similar wavelength, and also differing in amplitude. Looking at only the combined wave, it is much less obvious that the contributing waves were sinusoidal. Part (c) shows the same combination with just one difference: the relative position of the two waves. This change makes a very significant change in the shape of the superposition.

If you have read Chapter 37, you may be feeling déjà vu. This is, indeed, exactly the same superposition rule as for vibrations. The only differences are the labels on the axes. The vertical axis for vibrations was displacement, but here it can be any kind of disturbance, as appropriate for the wave. The horizontal axis for vibrations was time, but for these snapshot graphs of waves the horizontal axis is position.

If we wish to add the time behavior of the waves, we need to consider Figure 138.1 as one frame in a movie. In this movie, the gray contributing waves would slide sideways. If they were comoving, then they would slide in the same direction, as a group, with the result that the superposition (black curve) would also slide in the same direction, maintaining the same shape. If the contributing waves were countermoving, then as they slide in opposite directions the shape of the superposition would change. Figure 138.1(b) and (c) could be different frames taken from the same movie! Very likely, it would be difficult to describe the superposition as moving either leftward or rightward.

In order to draw a superposition by hand, you must at least find the sum at each maximum and minimum of every contributing wave. For example, in Figure 138.1(b) that would be the 12 positions marked by small arrows underneath. In part (c) 18 points would be necessary. Given these points, smooth connections between them will at least have the correct general shape. Of course, if the combination has a repeating pattern, as in Figure 138.1(b) and (c), that can cut down the required effort considerably.

It is especially easy to do the sum at positions where one of the contributing waves is at zero disturbance, such as position (iv) in Figure 138.1(b). At such points, the superposition curve crosses the non-zero contributing curve. Notice especially that when the contributing curves cross, as at position (v), the combination does *not* go through the intersection.

Chapter 139. 1D Superposition with Phase

When looking at the superposition of two sinusoidal waves, how they combine at each position is related to the difference between their phases at that position. Figure 139.1 shows two of the superpositions from Figure 138.1 with labels added for the reduced phase of each contributing wave, and also arrows showing them to be comoving. Phases for oscillations A (the solid gray lines) are above the graphs and phases for oscillations B (the dotted lines) are below. The phase increases in the direction opposite the direction of wave travel, as described in Chapter 122.

Each position where their reduced phases match is marked with a small arrow above the graph. At those points, the (reduced) phase difference between the contributing waves is zero,

$$\Delta\phi = \phi_A - \phi_B = 0° \quad , \tag{139.1}$$

and the two contributing waves are said to be **in phase.** The significance is that when they are in phase, the two sinusoids are "doing the same thing," and that thing is emphasized in the superposition. In graph (a) at the small arrows from above, both waves are at phase 0°, reaching a maximum, so the superposition displacement is especially large. In graph (b) at the small arrows from above, they are both at phase 90°, decreasing steeply with increasing x, so that the superposition is decreasing with an especially large negative slope. The large displacement is perhaps easier to notice than the large slope, but they are both examples of the reinforcement that occurs when waves are in phase.

Conversely, the arrows below the graphs in Figure 139.1 indicate where the (reduced) phase difference is 180°, and the vibrations are said to be **out of phase**. At those positions the two contributing waves are "doing opposite things," and tend to cancel each other. In graph (a), at the small arrows from below, one is at a minimum (i.e., negative density change) while the other is at a maximum, so that the superposition density change is relatively small (although not zero, because the amplitudes differ). In graph (b), at the small arrows from below, the contributing slopes are opposed, resulting in a smaller slope (in this case, zero) for the superposition.

The concepts of **in phase** and **out of phase** are most useful when the two contributing waves have similar wavelengths. Otherwise, the phase difference $\Delta\phi$ between them varies too rapidly with position. Figure 139.1 is useful for explaining the significance of phase relationships, but it does not show situations where the phase relationship is particularly useful for understanding superposition. That will come in other chapters.

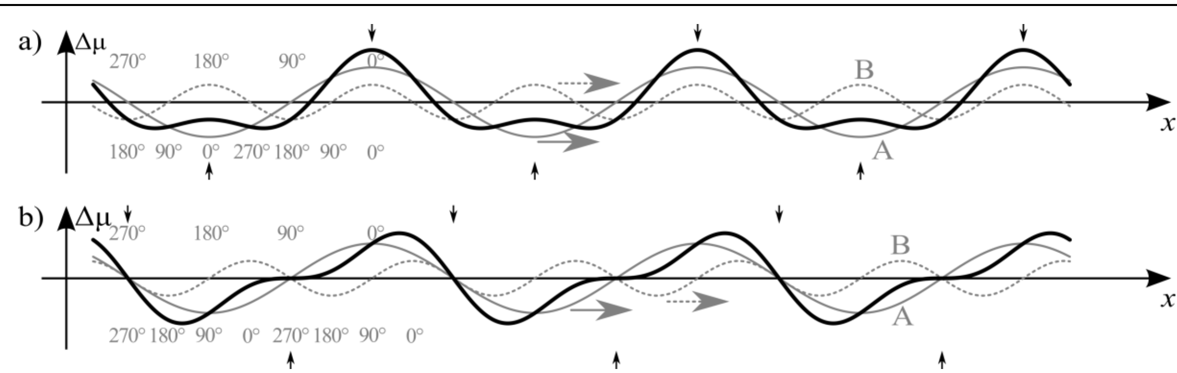

Figure 139.1

Superpositions of sinusoidal waves, highlighting phases and phase differences.

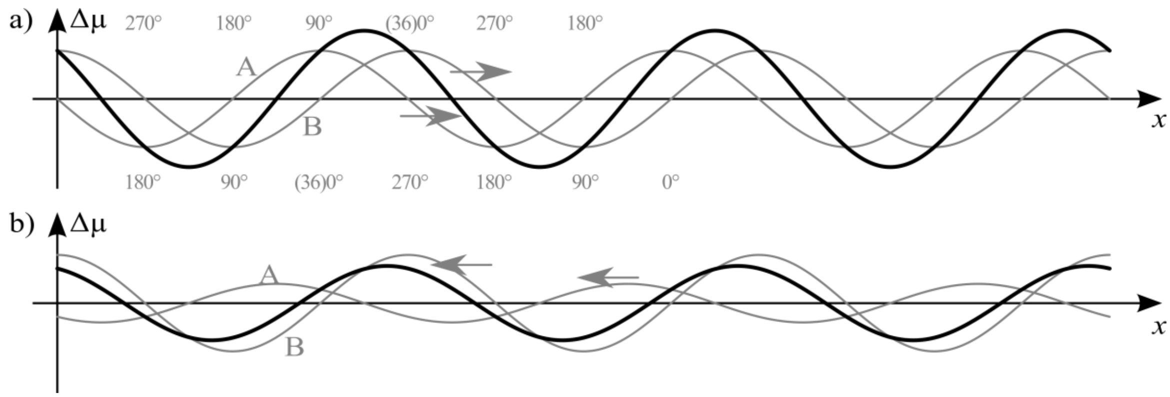

Figure 140.1

Superposition of two sinusoidal waves (gray) with same wavelength. Result is always another sinusoid (black). Arrows indicate the direction of wave travel, the same for both contributing waves. (a) Some phases of contributing waves are labeled; (36)0° represents that 360° is the same as 0°.

Chapter 140. Comoving Interference

The superposition of waves with the same frequency results in some special behavior, and therefore has the special name **interference.** (Chapter 39 gives the same name to the superposition of two vibrations with the same frequency.) Notice that since the two waves are traveling through the same medium, they must have the same wave speed. If they have the same frequency, then by Eq. 121.2 they must also have the same wavelength.

One specific example of wave interference is when sinusoidal one-dimensional waves are comoving. Figure 140.1 illustrates that this *always* results in a combination that is also a sinusoidal wave. Chapter 138 acknowledges that superposition of comoving 1D waves can seem a little artificial. Here it seems even more artificial: if the combination is a sinusoidal wave, then why consider it as a combination of two other waves? But there are some real-world situations for which this is a useful perspective, such as when two sources send sounds in the same direction and they then combine. Comoving interference also provides a nice stepping-stone to more complicated interference situations.

Figure 140.1(a) labels the reduced phase of many points on each of the two contributing waves. Recall from Chapter 122 that, in a snapshot graph, phase increases the direction opposite of wave travel. An exercise for the reader: which wave matches to the phases at the top, and which to the phases at the bottom?[42]

Since both contributing waves go through 360° of phase in the same distance (one wavelength), their phase difference is the same at any position *x* you choose,

$$\Delta\phi = \phi_B(x) - \phi_A(x) = \text{constant} \quad . \qquad (140.1)$$

In Figure 140.1(a) the phase difference is always 90°. In Figure 140.1(b) the phase difference equals a different constant, even though the two amplitudes are different. For two comoving sinusoidal waves, the entire waves can be in phase or out of phase, instead of just specific positions being so.

The name interference presumably comes from the fact that if the contributing waves are close to out of phase, then the superposition amplitude is smaller than at least one of the contributing amplitudes. Perfectly out of phase is the relationship that yields the smallest possible superposition amplitude, given by

[42] Answer: The top phases are for wave B, and the bottom phases are for wave A.

$$x_{\mathrm{m,sup}} = |x_{m1} - x_{m2}| \quad . \tag{140.2}$$

This situation is called **destructive interference.**

But superposition of equal-frequency waves is still called interference even when the waves are close to being in phase. When the contributing waves are exactly in phase, the superposition amplitude is the largest possible and is given by

$$x_{\mathrm{m,sup}} = x_{m1} + x_{m2} \quad . \tag{140.3}$$

This situation is called **constructive interference.**

It turns out that as long as the phase difference is between $-90°$ and $90°$ (that is, the waves are within a quarter cycle of being in phase), then the superposition wave amplitude will be larger than either amplitude of the contributing waves. Conversely, a phase difference between 120° and 240° (that is, within 1/6 cycle of being out of phase) ensures partial cancelation, with the superposition wave amplitude smaller than the larger of the two contributing waves.

If you have read Chapter 39, all this may seem very familiar. Indeed, the rules and the terminology are all the same as when combining vibrations of the same frequency. The only difference is how the graphs relate to time. In Chapter 39 time was explicitly shown on the horizontal graph axis. In this case, the graphs are snapshots, and if time were to advance then the waves would slide sideways along the *x*-axis.

Chapter 141. Source Displacement

The pairs of gray sinusoidal waves Figure 140.1(a) and (b) might come from two sources. One reason for the waves to be not in phase is if the vibrations of the sources are not in phase. The phase difference between the source vibrations is one factor in the phase difference between the waves.

But even two in-phase sources can produce waves that are not in phase, as illustrated in Figure 141.1. The vibrational motions of those sources are in phase. At the instant that the snapshot graphs represent, both are emitting the 0° phase of their waves, and one quarter of a period later they will both be emitting the 90° phase. But with the sources separated by a distance L, the waves have a phase difference equal to the phase change as the wave from A travels to reach source B. In fact, that phase difference is proportional to the separation, so that the phase difference can be calculated with the equation

$$\frac{\Delta\phi}{L} = \frac{360°}{\lambda} \quad , \tag{141.1}$$

$$\Delta\phi = \phi_B - \phi_A \tag{141.2}$$

where $\Delta\phi$ is the phase of wave B **relative to** wave A. As in Eq. 140.1, ϕ_B and ϕ_A are the phases of waves A and B at a specific location x, but the result is the same regardless of which location is used. In the

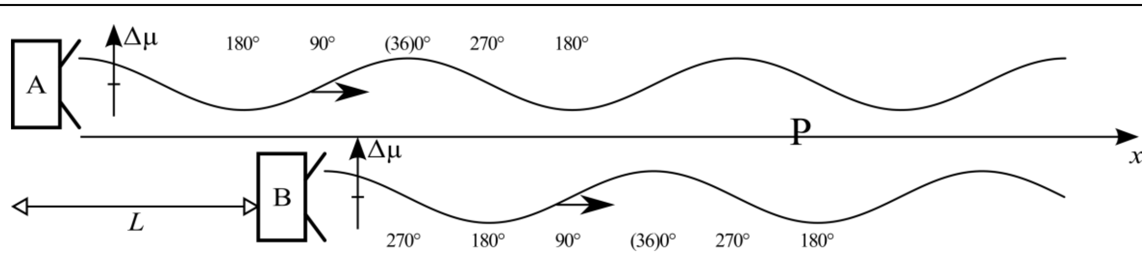

Figure 141.1

Two speakers producing comoving sound waves.
Even though speakers are vibrating in phase, waves are not in phase.

figure, the separation is $L = \frac{3}{4}\lambda$ so that wave B is 270° ahead of wave A (which is the same as 90° behind wave A, since 360° and 0° are the same phase). If the separation were an integer number of wavelengths, the waves would once again be in phase.

There's another way to think of the distance L. To reach the place marked P in Figure 141.1, L is the extra distance that the wave from source A must travel, compared to the wave from source B. Both waves go through a certain phase change traveling from the source to point P, but the parts of travel which are the same for both waves don't result in a phase difference. It is only the *extra* distance traveled that has an impact. If L_A and L_B are the distances from the respective sources to point P, then Eq. 141.1 can be written

$$\frac{\phi_B - \phi_A}{L_A - L_B} = \frac{360°}{\lambda} \quad , \tag{141.3}$$

Of course, this would be true regardless of where we placed P. We have already noted that the phase difference between comoving waves is the same everywhere that the waves are combining. But this focus on position P puts a different spin on that thought, by relating to the wave sources.

In Eqs. 141.1–141.3, the length $L = L_A - L_B$ might be larger than the wavelength, so that the corresponding phase difference $\Delta\phi$ is greater than 360°. In this situation it is easiest *not* to reduce the phase difference into the 0° to 360° range.

Chapter 142. Traveling Wave Energy

Traveling waves carry disturbances through media. Energy is required to create disturbances, working against the restoring force. Therefore, it is one of the defining characteristics of waves that they contain energy, and that traveling waves transport that energy. The connection is so strong that the definition in Chapter 3 says that sound *is* "mechanical radiant energy." Some people might even say that moving energy through a nearly stationary medium is the primary characteristic of all waves, and that the disturbance of a medium is a secondary feature.

The repetitive nature of periodic sources and periodic waves assures us that, in periodic cases, the same amount of energy is associated with each cycle. Each cycle can be thought of as a packet of energy. Although, that is not meant to imply that the energy is bunched up inside that cycle.

For waves through material media, the energy transported is mechanical energy. Chapter 29 describes how different forms of mechanical energy can be categorized as either kinetic or potential, and the following chapters will address those in turn. Exactly where in a wave is each of these types found? Transverse one-dimensional waves are considered first, and then the conclusions are extended to compression waves.

Chapter 143. Wave Potential Energy

143a. Transverse Wave Potential Energy

Recall that when a mass and spring system has potential energy, that potential energy is contained in the spring, not in the mass. But for a transverse wave on a rope, when a short piece of the rope is displaced, there is no spring pulling that piece back toward its equilibrium position. The restoring force, the tension in the rope, is more indirect.

For a truly transverse wave, pieces of the medium move exactly perpendicular to the wave travel direction. In Figure 143.1 the rope material between positions (i) and (iv) is the same for time (a) as it is for time (b). But the length of line between those positions is clearly longer at time (b). So, a truly transverse 1D wave requires that the medium be stretchy. In Figure 143.1, it may look absurdly stretchy for a real rope. Remember that snapshot graphs may not have the same vertical and horizontal scales, so that the stretching may be much less than it appears. However, it is also true that waves on real ropes are often not perfectly transverse.

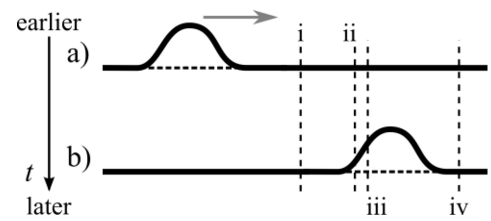

Figure 143.1

Transverse pulse moves along a rope. Positions i–iv mark locations in the medium.

In any case, there is some stretching of the medium, and just as with a spring, the potential energy lies where the medium has been stretched. In Figure 143.1(b), between positions (ii) and (iii) is a section of the rope which has been maximally stretched, because the originally horizontal piece is now covering a longer diagonal. In general, the potential energy is located in places where the snapshot graph is not parallel to the equilibrium shape. Displacement of the medium does not itself contain potential energy—at the maximum displacement in Figure 143.1, the rope is parallel to the equilibrium shape, and hence is not stretched at all. Instead, the potential energy is in places where a snapshot shows a varying displacement along the rope. For rope waves that are not perfectly transverse, the potential energy will still be concentrated near such places.

143b. Compression Wave Potential Energy

For a longitudinal/compression wave, the same conclusions apply, but it is perhaps clearer to consider the compression and stretching of the medium directly. When the wave is considered as a variation in density, the potential energy will be greatest in the places where the density is farthest from equilibrium.

To see how this is in agreement with the rule for transverse waves, a look at Figure 124.2 is helpful. Comparing Figure 124.2(c) and (d), the extremes of density are indeed the places where the displacement snapshot graph has the greatest slope.

Chapter 144. Wave Kinetic Energy

144a. Transverse Wave Kinetic Energy

Figure 144.1 shows two transverse wave pulses on a rope, each one carrying energy. When they arrive in the same location at time (c), the prescription of superposition results in the two pulses "canceling out." Where has the energy gone? Conservation of energy requires that it has not disappeared. The conclusion from Chapter 143 suggests that (c) has no potential energy. In Chapter 29, kinetic energy is called the most visible form of energy. That is true when watching an event in real life. But because kinetic energy is revealed by motion, it is completely invisible in a photograph or snapshot graph. This is where the energy is hiding in Figure 144.1(c).

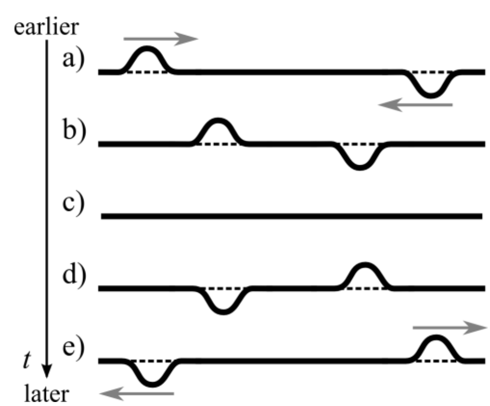

Figure 144.1

Two oppositely moving wave pulses, with same shape but opposite sign displacement. Five snapshots from equally spaced moments in time.

To find this kinetic energy, we must look more closely, and at several sequential snapshots. Figure 144.2 shows the same crossing of two wave pulses, but at times that are much closer together, and zoomed in so that the pulses look wider. Focus your attention on the two positions indicated by dots. Considering the positions of those points before and after reveals that at time (c) they have the velocities indicated by the arrows.

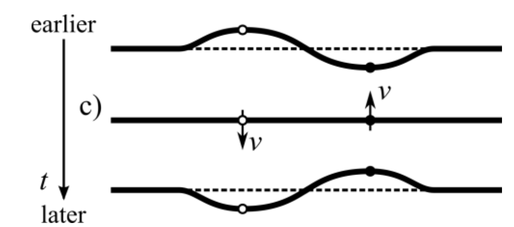

Figure 144.2

Magnified view of center of Figure 144.1(c), along with moments just prior and after that time.

The wave speed is not (directly) involved with this kinetic energy. The kinetic energy formula

$$KE = \frac{1}{2}mv^2 \tag{144.1}$$

requires something with mass to have the speed. A wave has a speed, but it has no mass of its own. The wave transports energy *of the medium*, and in this case the medium has velocity in a transverse direction.

Using these ideas, we can look in more detail at where a traveling wave carries its kinetic energy. First consider the transverse wave pulse on a string shown in Figure 144.3 by a few superimposed snapshots. During the short time interval depicted, position (b) has an upward velocity while position (a) has a downward velocity. As the positive displacement passes through a point in the medium, that point is first moved in a positive transverse direction, and then moved back in a negative transverse direction until it returns to the original, equilibrium point.

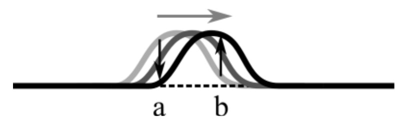

Figure 144.3

Several snapshots of a rope wave pulse, superimposed. Most recent snapshot is darkest, pulse is moving rightward.

The general lesson is that a single traveling transverse wave has its kinetic energy wherever a snapshot graph shows a varying displacement along the wave, or in other words, in places where the snapshot of the wave is not parallel to the equilibrium shape. Note that in Figure 144.1(c) and Figure 144.2(c) this general rule appears to be violated. That's because two countermoving waves are combining, which requires more careful consideration.

144b. Compression Wave Kinetic Energy

For compression waves, the same rule works with respect to the longitudinal displacement graph. Wherever the slope of the Δx vs. x graph is the greatest, as the wave passes through (whether leftwards or rightwards), those places will change position most rapidly. Changing displacement is exactly what gives velocity and kinetic energy. For example, in Figure 124.2 the maximum kinetic energy is where the displacement is zero, not because of the value of Δx, but because that is also where the graph is steep.

Comparing Figure 124.2(c) and (d), these are the same positions as where the density is farthest from equilibrium. This is something of a coincidence, as there is no direct link between density and kinetic energy. But it is a reliable rule that large density deviations from equilibrium imply rapidly changing displacement, and thus high kinetic energy.

Chapter 145. Combined Wave Energy

As a sinusoidal wave passes through a point in the medium, that parcel of the medium moves sinusoidally, just like a mass on a spring. The parcel reaches its maximum kinetic energy when it is moving the fastest, while it is passing through its equilibrium position, just like a mass on a spring. It is tempting to stretch this analogy further by assuming that the parcel has a maximum of potential energy when it is furthest from equilibrium. Unfortunately, that is taking the analogy too far.

As it turns out, we have found that in traveling mechanical waves the kinetic and potential energies are greatest in the same locations, and zero at the same locations. The total energy in a traveling mechanical wave is thus not uniformly spread throughout the wave, but instead is concentrated near places in a snapshot graph where the displacement most rapidly varies along the direction of wave travel. For sinusoidal waves, these are the places of zero displacement! Less surprisingly, for compression waves these are places of maximum magnitude of density disturbance.

What, then, are we to make of Chapters 49 and 50, in which the power and intensity of sound was modeled as if the energy in a sound wave was uniformly spread throughout the wave? We now have shown that model is wrong. The energy is instead concentrated into sheets, perpendicular to the direction of wave travel, with two sheets in each wavelength. However, the uniform energy model is still useful whenever dealing with lengths significantly larger than a wavelength, or times significantly longer than a period. In those situations, we can treat the energy as if it were uniformly spread, at a value equal to the average of the true distribution. The errors introduced will be small, and the calculations will be much easier.

Keep well in mind that these conclusions have applied to traveling waves. Standing waves move in a different way, so conclusions about kinetic energy in particular are not the same. The specifics are covered in Chapter 185. Here we will just note that care is required to keep the distinction in mind. Standing and traveling waves differ in how they move, but not in how their snapshots look, so a single picture or graph like Figure 124.2 can easily be mistaken for the wrong case.

Chapter 146. Waves at Boundaries

When a medium is uniform in its properties, the same everywhere, then waves travel in straight lines through the medium. (For one-dimensional waves, traveling along the medium is the only choice available!) But when a wave comes to a boundary of its medium, a place where the medium or its properties change abruptly, more options arise. The boundary might absorb all or some of the **incident** wave (the wave approaching the boundary), changing the wave's energy into some other form. This is what happens at the walls of a **dead room.** The most extreme example is an **anechoic chamber,** which is specifically designed so that sound reaching the walls is completely absorbed. On the other hand, some of the incident wave might be **reflected** back into the medium from which it came. And if whatever is beyond the boundary can host a wave, some of the wave might be **transmitted** to the other side of the boundary. This and the following chapters consider what happens if very little of the energy is absorbed.

What exactly qualifies as a boundary? Technically, it is an abrupt change in a property of the media called the **acoustic impedance** of the medium. But this book will not get into what impedance is. Instead, we'll just note that it is connected to the medium's massiveness, the restoring force, and the wave speed. Thus, a boundary can often be recognized by a change in one of those properties. For the changes of the medium properties to form a boundary, they must be abrupt changes. As a rule of thumb, this means that the changes in the media must happen over a distance that is quite a bit shorter than the distance over which the wave disturbance changes much. For a periodic wave, that means much shorter than one wavelength.

Suppose that a periodic incident wave reaches a boundary between two media which are **linear** in the sense used in Chapter 115. To have something specific in mind, this could be a transverse wiggle in a rope, which is tied with a knot to a second thinner rope. As each cycle of the wave reaches the junction, the knot moves in some way, and a wiggle is transmitted into the second rope. Even without knowing anything more, it is certain that the results will also be periodic, as each incoming cycle has the same effect on the junction. The period and frequency of the transmitted wave will equal that of the incident wave.

There may also be a reflected wave, carrying energy back into the first, thicker rope. This might make the motion of that rope a little crazy, because the incident wave is present there as well. (See Chapter 177 for some idea of how that might look for sinusoidal waves.) But with a nondispersive, linear medium, we know that the two waves function independently. The same thinking from above applies to the reflection: whatever the incident wave does at the boundary, it must be periodic. The period and frequency of the reflected wave will equal that of the other two.

The reflected wave is in the same medium as the incident wave, and therefore travels at the same speed (even though in a different direction). With Eqs. 121.1 and 121.2, the reflected wavelength must then also be the same as the incident wavelength. This might be difficult to discern near the boundary, due to interference between the waves. But if the incident wave stops, then eventually the echo will separate from the initial wave, these parameters could be measured.

The transmitted wave likely has a different speed from the incident wave, since it is in a different medium. Equations 121.1 and 121.2 show that because of the constant frequency, the transmitted wavelength is proportional to the new speed. A speed-up will stretch the wavelength, and a slow-down will contract it.

Chapter 147. Reflection at Boundaries

A reflection can occur at any boundary. But first, consider what happens when the boundary is the end of the medium, and there is nothing beyond the end that can carry a wave. In this case, any incident wave must reflect when it reaches the boundary. The energy in the disturbance must go somewhere. If there is no absorption, then returning into the original medium is the only option.

However, that reflected energy can be realized in different ways. As the wave is reflected, it can also change shape. It would be very difficult to see this with a continuous incident wave. Instead, Figure 147.1 shows various wave pulses reflecting from ends of strings, with two very different cases. In parts (c) and (d) the end of the string is **fixed** in place, completely unable to move, represented by being attached to a large block. In parts (a) and (b) the end of the string is **free** to move up and down, represented by a ring that can slide on a post. (The end of the string cannot be completely free, unattached to anything, because then there would be no way to maintain the restoring force that's required for a wave to travel.)

As each bit of the wave pulse shape reaches the boundary, that bit reflects and starts to move back into the rope. For example, the small leading bump reflects off the end first, and remains in the lead, ahead of the larger bump, after the reflection has changed the direction of travel. As a result, the reflection causes the wave pulse shape to **reverse** left for right in all cases.

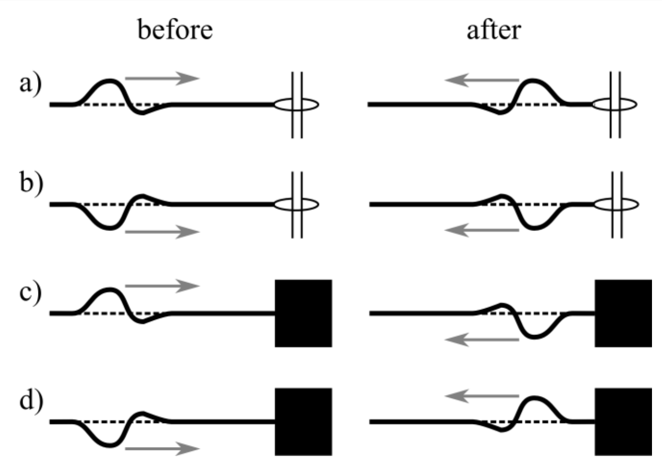

Figure 147.1

Wave pulses on string reflecting from end of the string. (a & b) Pulses reflect off free end. (c & d) Pulses reflect off fixed end.

In parts (a) and (b), showing reflection from a **free end**, the larger displacement keeps the same (vertical) sign throughout the reflection. In part (a) the upwards displacement remains upward after the reflection, and in part (b) the larger displacement is downwards both before and after the reflection. This is called remaining **upright.** This term may seem a little misleading in the case of a downward pulse, when "remaining upright" means "continuing to be downward."

In parts (c) and (d), showing reflection from a **fixed end**, instead of remaining upright the pulses **invert,** or flip top for bottom. It is still true that they **reverse** as well. Notice that two very similar words are being used for two different things: **reverse** means to flip along the direction of travel, and **invert** means to flip perpendicular to the direction of travel.

This inverting behavior may seem very non-intuitive. Since it is easier to learn something when you can make sense of it, consider this story. If an upward pulse were to approach the wall, then in order to keep the end fixed, the wall must apply a downward force on the string. That sudden downward force "cancels out" the incident upward pulse, but it has a side effect: it creates the new, reflected pulse. Thus, the reflected pulse is downward, inverted from the incident pulse.

Chapter 148. Compression Waves at Boundaries

The examples of Figure 147.1 were of transverse waves, where the disturbance in the medium is best understood as a displacement. The same rules apply to other types of waves, but in ways that are worth explicit consideration. For sound, we are particularly interested in compression waves. In order to continue to focus on one-dimensional waves, and also because of its importance to wind instruments, we'll look at compression waves traveling down a tube.

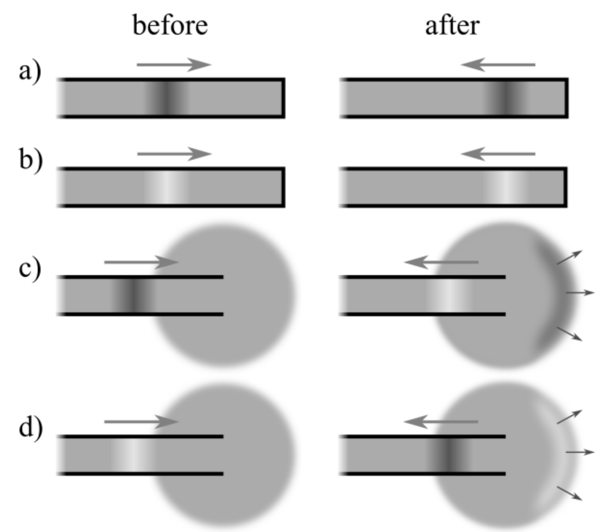

Figure 148.1

Pressure pulses in air in a tube reflecting from tube end. Darker gray shows overpressure, lighter gray shows underpressure. (a & b) Pulses reflect off closed end. (c & d) Pulses reflect off open end, air outside also plays a part.

When a compression wave pulse travels along a tube and reaches a **closed end**, as in Figure 148.1(a) and (b), a reflection will result. The energy in the disturbance has nowhere else to go. It might be a surprise, however, that an **open end** of a tube also causes a reflection. In Figure 148.1(c), when the compression pulse reaches the end, why shouldn't it simply continue into the outside air? Indeed, some of the pulse is transmitted to the outside air, but some of the wave energy is also reflected back along the tube. How this arises out of more basic physics is beyond the level of this book. As a way to make it more intuitively sensible, you can imagine that as the slug of extra air exits the tube, it overcompensates by sucking more air out of the tube than it should, creating the reflected rarefaction pulse.

In Figure 148.1, parts (a) and (b) show that whether the incident wave pulse is a compression or a rarefaction, the pulse reflected from a closed end is of the same type. Parts (c) and (d) illustrate that reflecting from an open end inverts a compression into a rarefaction and vice versa. Comparing to Figure 147.1, it seems that an open tube end is like a fixed end of a string (causing inversion), and a closed tube end is like a free string end. Again, this might be a surprise. How can an *open* end be like a *fixed* end? An open end seems like freedom! Conversely, at a *closed* tube end does not seem like the air is *free*.

The resolution to the quandary is this: Whether an end is fixed or free must be considered from the same perspective that is used to describe the wave disturbance. A closed tube end does indeed fix the position of the air at the end, preventing displacement. If we were to graph the longitudinal displacements of these waves, the closed end would indeed invert the pulse. Similarly, an open tube end freely allows displacement, and a graph of longitudinal displacements in the cases of Figure 148.1(c) and (d) would show the pulse remaining upright.

But from a compression perspective, the open end is (mostly) fixed, because it tries to keep the air density the same as the air density outside of the tube. At a closed end, the density is free to increase, when the air crowds up against the end, or to decrease, when the air is sucked away from the end.

> Density is free to change at a closed tube end, so that compression waves remain upright upon reflection.

> Density is fixed at an open tube end, so that compression waves invert upon reflection.

This difference between the displacement and compression viewpoints is closely related to the sideways shift observed between the graphs in Figure 124.2(c) and (d). There, the largest displacements occurred where the compression was zero, and vice versa. Here we have seen that fixed displacement ends are free compression ends, and vice versa.

Chapter 149. Transmission at Boundaries

149a. Transmission of Wave Energy

If a medium beyond a boundary can carry a wave, then some of the wave energy can be transmitted into that second medium. But it remains true that some of the wave energy can be reflected from the boundary. This section is focused on the balance between these two options.

Figure 149.1 details the range of possible cases. The diagrams across the top illustrate the cases with the specific example of a transverse wave on a string. In each diagram the **incident wave** would approach the boundary from the left, symbolized by the arrowhead. In the middle three diagrams, the boundary attaches to a second medium on the right, into which some of the incident wave can be transmitted. The leftmost and rightmost diagrams show the same cases described in Chapter 147, with no second medium. They can be considered extreme examples of the range of possibilities.

A wave on a string is one of the common cases where the restoring force does *not* change across the boundary. The acoustic impedance changes only due to the density, and is apparent in the relationship between the wave speeds of the incident wave (s_i) and the transmitted wave (s_t). On the left half of the figure, the medium of the transmitted wave is lighter than the incident wave medium, so the transmitted wave speed is higher. On the right half, those relationships are reversed. The whole figure represents a continuum of possibilities.

The lower part of Figure 149.1 shows what will occur after the wave has reached the boundary. In general, the wave energy will split into two parts, some of it transmitted and some of it reflected. This is represented by the balance between gray and white, respectively above and below the diagonal lines. Only in the center case, where two equivalent media are joined, will the wave be completely transmitted with 0% reflection. Moving away from the figure's center in either direction, as the mismatch between the two media gets

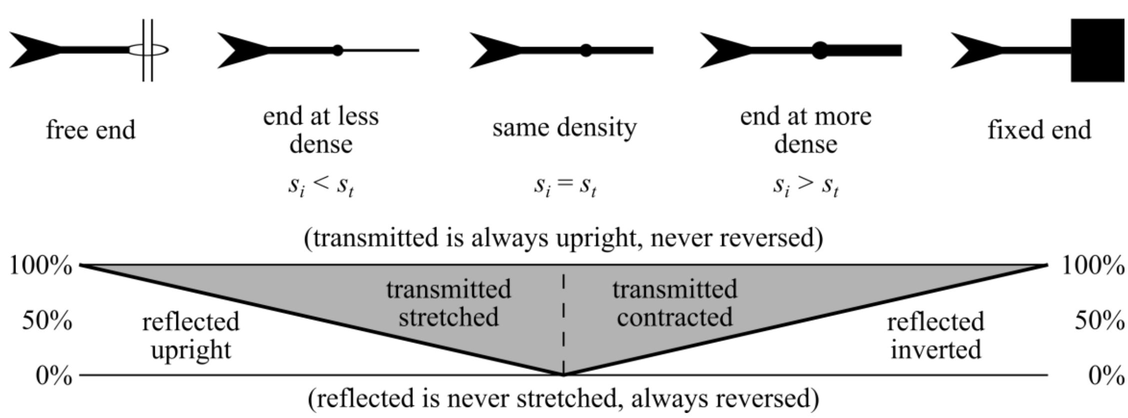

Figure 149.1

Range of possibilities for boundaries between two media. Top row illustrates them as a rope (left) attached to various things on right; arrowhead represents incident wave direction. Bottom graph shows fraction of incident wave energy reflected; remainder (gray, above the line) is transmitted.

greater more of the energy is reflected, as indicated by the vertical scale. This leaves less to be transmitted. At the extremes, if the second medium simply isn't there (left side) or is too massive to have a wave in it (right side), all of the wave energy is reflected.

149b. Shape Changes

The changes in reflected wave shape described in Chapter 147 also apply in the intermediate cases. Reflections are always reversed. A lighter transmission medium (left half of Figure 149.1) means that the junction is easier to move, and the reflected wave remains upright. This is illustrated in Figure 149.2(a). Reflections are inverted only when there is a heavier medium beyond the boundary (right half of Figure 149.1). This inversion is illustrated in Figure 149.2(b). A fixed end is the extreme case of this, the ultimate example of "heavier." Notice a surprising fact: whether these reflected waves invert or remain upright depends on the second medium, *which the reflected waves never enter!*

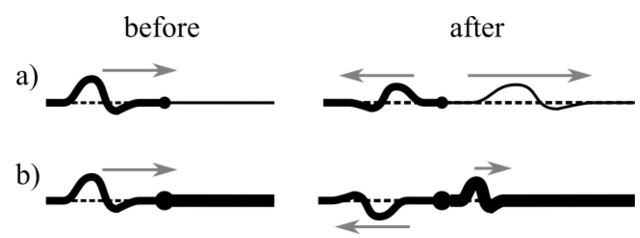

Figure 149.2

Wave pulses on string transmitting into another medium. Second medium has (a) faster wave speed, and (b) slower wave speed.

Transmitted waves are never reversed nor inverted. The incident shape translates directly into the new medium, bit by bit. However, another type of shape change can occur. If the new medium has a different wave speed, then the front of a wave pulse moves at that speed as soon as it crosses the boundary. While the pulse is crossing the boundary, the front and back of the pulse have different speeds, and as a result, the wave can be stretched or contracted. Figure 149.2 shows the two general cases, which are also labeled in Figure 149.1. These are just the pulse wave equivalents of the wavelength changes noted in Chapter 146.

When compression waves are transmitted from one medium to another, the same rules apply. However, media for compression waves are more likely to have boundaries where the restoring force changes, which makes it harder to identify which case applies out of the range of options in Figure 149.1. For example, when a sound reaches a wall and some of it enters the solid wall (perhaps on the way to the neighboring room), the reflecting part of the sound remains upright (as with the closed tube end in Chapter 148) even though the density increases from air to wall. These are situations for which an understanding of impedance is important, and so they are beyond the level of this book.

Figure 148.1(c) and (d) provide other examples where a compression wave is both reflected and transmitted. In that situation it is hard to recognize that a boundary even exists, since the density and wave speed are the same everywhere. Somehow there is an impedance change with no speed change. Since the reflected pulse inverts, it appears that a compression wave reaching an open tube end reflects similarly to the right half of Figure 149.1. However, since the wave speed doesn't change, the transmitted wave is not contracted.

Chapter 150. Acoustic Impedance

This book does not describe what **acoustic impedance** is or how it is used. But a bit more about it may be useful for recognizing boundaries.

Each particular material has as one of its properties a **specific acoustic impedance.** There are general patterns relating the specific impedance to other properties of the medium, including the following

$$\text{specific impedance} = (\text{massiveness})(\text{wave speed}) \quad , \tag{150.1}$$

$$\text{specific impedance} = \sqrt{(\text{massiveness})(\text{restoring force})} \quad , \tag{150.2}$$

$$\text{specific impedance} = \frac{(\text{restoring force})}{(\text{wave speed})} \quad . \tag{150.3}$$

These three relationships are interrelated by the wave speed pattern in Eq. 116.1, but it is useful to have them all available for consideration. These are most revealing when one of the three quantities is the same on both sides of the boundary. For example, suppose there is a boundary across which the restoring force doesn't change, the medium density increases, and the wave speed decreases. Then Eq. 150.1 is not as useful, because we don't know which change (density or speed) is more significant. However, either Eq. 150.2 or Eq. 150.3 show that the impedance increases. That conclusion is possible without needing to know exact numerical quantities.

Beyond this, a specific shape of medium has an **acoustic impedance,** which depends on both the specific impedance and the shape. Figure 148.1(c) and (d) show situations where the medium is the same everywhere, so the specific impedance is also the same everywhere. However, the impedance is different inside and outside of the tube, only because of the walls of the tube, which define the shape of the air inside. This is why the end of the tube qualifies as a boundary, from which there can be partial reflection of a wave.