

Handout: Relationship between distributed load, shear force, and bending moment

Find the shear force and bending moments in the beam having the loading shown.

First, find $w_{AB}(x)$: $w_{AB}(x) = w_{\max}(1 - x/L_{AB})$

Find the shear force for section AB:

$$V_{AB}(x) = -\int w_{\max} \left(1 - \frac{x}{L_{AB}}\right) dx$$

$$V_{AB}(x) = -w_{\max} \left(x - \frac{x^2}{2L_{AB}}\right) + C_1 \quad \text{but } C_1 = 0, \text{ so}$$

$$V_{AB}(x) = -w_{\max} \left(x - \frac{x^2}{2L_{AB}}\right)$$

Find the V_B to use as a boundary condition for the next region:

$$V_B = -w_{\max} \left(L_{AB} - \frac{L_{AB}^2}{2L_{AB}}\right) = \frac{-w_{\max} L_{AB}}{2} \rightarrow \boxed{V_B = -45\text{N}}$$

Find the shear force for section BC. In this region, $w = 0$:

$V_{BC}(x) = -\int 0 dx = C_2 = V_B$ because if V_{BC} is constant, then it must be the same as point B which we already know.

$$V_{BC}(x) = \frac{-w_{\max} L_{AB}}{2}$$

Find the bending moment for section AB:

$$M_{AB}(x) = \int -w_{\max} \left(x - \frac{x^2}{2L_{AB}}\right) dx = -w_{\max} \left(\frac{x^2}{2} - \frac{x^3}{6L_{AB}}\right) + C_3 \quad \text{But, } M \text{ at point A is zero, so } C_3 = 0:$$

$$M_{AB}(x) = -w_{\max} \left(\frac{x^2}{2} - \frac{x^3}{6L_{AB}}\right)$$

Find the M_B to use as a boundary for the next region: $M_B = -w_{\max} \left(\frac{L_{AB}^2}{2} - \frac{L_{AB}^3}{6L_{AB}}\right) = \frac{-w_{\max} L_{AB}^2}{3} \rightarrow \boxed{M_B = -90\text{Nm}}$

Finding the bending moment for section BC:

$$M_{BC}(x) = \int \left(\frac{-w_{\max} L_{AB}}{2}\right) dx = \frac{-w_{\max} L_{AB} x}{2} + C_4 \quad \text{Using } M_B \text{ at } x=L_{AB}:$$

$$\frac{-w_{\max} L_{AB}^2}{2} = \frac{-w_{\max} L_{AB} L_{AB}}{2} + C_4 \rightarrow C_4 = \frac{+w_{\max} L_{AB}^2}{2}$$

$$M_{BC}(x) = \frac{-w_{\max} L_{AB} x}{2} + \frac{w_{\max} L_{AB}^2}{2} \quad \boxed{M_{BC}(x) = \frac{w_{\max} L_{AB}}{6} (L_{AB} - 3x)}$$

The shear and bending moment diagrams are shown:

