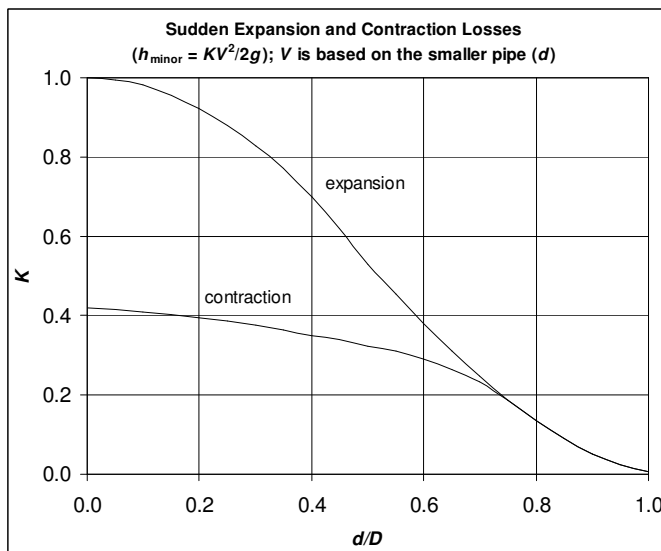
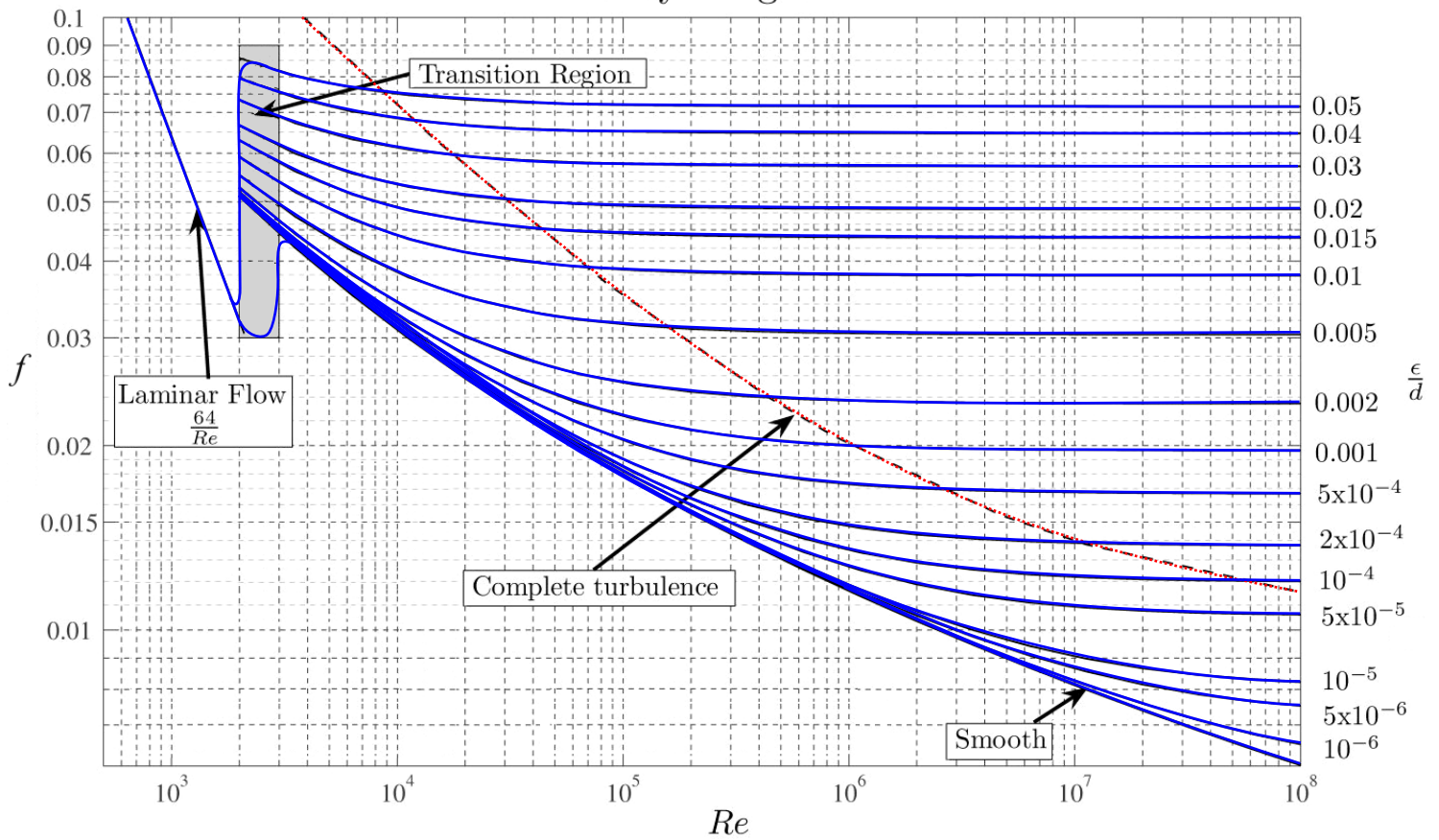


Pipes

Moody Diagram



Entrance loss coefficient: $K = 0.5$ (sharp edge)
Exit loss coefficient: $K = 1.0$ (any edge shape)

Roughness Values for Ducts:

Material	Condition	e (mm)
Steel	Sheet metal, new	0.05
	Stainless, new	0.002
	Commercial, new	0.046
	rusted	2.0
Iron	Cast, new	0.26
	Wrought, new	0.046
	Galvanized, new	0.15
Brass	Drawn, new	0.002
Plastic	Drawn tubing	0.0015
Glass	-	Smooth
Concrete	Smoothed	0.04
	Rough	2.0
Rubber	Smoothed	0.01

Basics

$$g = 9.8 \text{ m/s}^2$$

$$\rho_{\text{water}} = 998 \text{ kg/m}^3$$

$$P_{\text{atm}} = 101.3 \text{ kPa}$$

$$\mathbf{V} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$$

$$\tau = \mu du/dy$$

$$\nu = \mu/\rho$$

$$\text{SG} = \rho/\rho_{\text{H}_2\text{O}}$$

$$\text{Ideal gas: } P = \rho R_{\text{gas}} T$$

$$\vec{F} = -\int p d\vec{A}$$

$$P_{\text{gage}} = P_{\text{abs}} - P_{\text{atm}}$$

$$\vec{x} F_{\text{total}} = \sum x_i F_i = \int x dF$$

Hydrostatics

$$\vec{\nabla} p = \rho \vec{g}$$

$$p_2 - p_1 = \rho g(z_1 - z_2)$$

$$F_{\text{buoyant}} = +\rho_w g V_{\text{submerged}}$$

$$W = +\rho_o g V_{\text{TOT}}$$

$$\rho_o/\rho_f = V_{\text{submerged}}/V_o$$

$$-\vec{\nabla} p + \rho(\mathbf{g} - \mathbf{a}) = 0$$

$$\mathbf{g}_{\text{eff}} = \mathbf{g} - \mathbf{a}$$

Boundary Layers

$$\frac{\delta}{x} \approx 5.0 \left(\frac{\nu}{Ux} \right)^{0.5} = \frac{5.0}{\text{Re}_x^{0.5}} \quad (\text{Blasius: laminar})$$

$$\frac{\delta}{x} \approx \frac{0.16}{\text{Re}_x^{1/7}} \quad (\text{turbulent})$$

Pipe Flow

$$\left(\frac{P_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 \right) - \left(\frac{P_2}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 \right) = \frac{(u_2 - u_1)}{g} - \frac{1}{g} \frac{\partial Q_{in}}{\partial m} = H_{LT} = H_{LM} + H_{Lm}$$

$$H_{LM} = f \frac{L \bar{V}^2}{2gD}; \quad H_{Lm} = k \frac{\bar{V}^2}{2g}$$

$$f_{\text{LAM}} = 64/\text{Re}_D$$

$$\frac{1}{\sqrt{f}} \approx -1.8 \log \left[\frac{6.9}{\text{Re}_D} + \left(\frac{\epsilon/D}{3.7} \right)^{1.11} \right]$$

$$u = u^* \left(B + \frac{1}{\kappa} \ln \left(\frac{yu^*}{\nu} \right) \right)$$

where $u^* = \sqrt{\tau_{\text{wall}}/\rho}$,
 $B = 5$, and $\kappa = 0.41$

$$D_H = \frac{4A}{\text{Perimeter}}$$

Dimensional Analysis

$$\text{Re} = \rho UL/\mu$$

$$\text{Fr} = V^2/gh$$

$$C_D = F/(\frac{1}{2} \rho A V^2)$$

$$\Pi \text{ theorem: } (n - m) \Pi \text{ groups.}$$

Bernoulli

$$p_0 = p_{\text{dyn}} + p_{\text{stat}}$$

$$p_{\text{dyn}} = \frac{1}{2} \rho V^2$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

Integral Conservation Laws

$$M = \int_V \rho dV$$

$$\int_{CM} \vec{F}_{\text{external}} dt = \int \rho \vec{V} dV$$

$$N = \int_{\text{volume}} \rho n dV$$

$$\dot{m} = -\int_A \rho \vec{V} \cdot d\vec{A}$$

$$Q = \int \vec{V} \cdot d\vec{A}$$

$$\dot{N} = -\int n \rho \vec{V} \cdot d\vec{A}$$

$$\left. \frac{dN}{dt} \right|_{CM} = \frac{d}{dt} \int_{CV} n \rho dV + \int n \rho \vec{V} \cdot d\vec{A}$$

$$0 = \frac{d}{dt} \int_{CV} \rho dV + \int \rho \vec{V} \cdot d\vec{A}$$

$$\Sigma F_{CV} = \frac{d}{dt} \int_{CV} \vec{V}_{xyz} \rho dV + \int \vec{V}_{xyz} \rho \vec{V}_{xyz} \cdot d\vec{A} + \int \vec{a}_{RF} \rho dV$$

$$\dot{Q}_{in} - \dot{W}_{\text{net out}} = \frac{\partial}{\partial t} \int_{CV} e \rho dV$$

$$+ \int_{CS} \left(u + pv + \frac{V^2}{2} + gz \right) \rho (\vec{V} \cdot d\vec{A})$$

Potential Flow

$$\zeta = \nabla \times \mathbf{V} = 2\boldsymbol{\omega}$$

Streamlines: $dy/dx = v/u$

Irrotational, 2D, steady: $\nabla^2 \psi = 0$

$$\psi = -\int v dx + f_1(y)$$

$$\psi = +\int u dy + f_2(x)$$

$$\Delta \psi = Q/b$$

$$\vec{V} = \vec{\nabla} \phi \rightarrow u = \partial \phi / \partial x, v = \partial \phi / \partial y$$

Irrotational, steady: $\nabla^2 \phi = 0$

$$\Gamma = \int \vec{\nabla} \times \vec{V} dA = \oint \vec{V} \cdot d\vec{s}$$

$$w(z) = \phi + i\psi, \text{ where } z = x + iy$$

$$\frac{dw}{dz} = v_x - iv_y$$

Lift: $L = w \rho U_{\infty} \Gamma_{\text{TOTAL}}$

Differential Conservation Laws

$$0 = \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V})$$

$$\vec{a}_{\text{particle}} = \frac{D\mathbf{V}_{\text{particle}}}{Dt} \equiv (\vec{V} \cdot \vec{\nabla}) \vec{V} + \frac{\partial \vec{V}}{\partial t}$$

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla}) \vec{V} = -\frac{1}{\rho} \vec{\nabla} p + \vec{g} + \frac{\mu}{\rho} \nabla^2 \vec{V}$$

Basic Frictionless Compressible Nozzle Information for Air

$$c_p = 1005 \text{ J/kgK} \quad c_v = 718 \text{ J/kgK} \quad R = 287 \text{ J/kgK} \quad k = 1.40 \quad R = c_p - c_v \quad k = c_p/c_v$$

Functions valid everywhere on either side of a standing shock:

$$p = \rho RT \quad a = \sqrt{kRT} \quad v_{\text{lim}} = \sqrt{2c_p T_0}$$

$$T^* = T_0 \left(\frac{5}{6} \right) \quad p^* = p_0 \left(\frac{5}{6} \right)^{3.5} \quad \rho^* = \rho_0 \left(\frac{5}{6} \right)^{2.5} \quad a^* = a_0 \left(\frac{5}{6} \right)^{0.5}$$

$$T_0 = T (1 + 0.2Ma^2) \quad p_0 = p \left(\frac{T_0}{T} \right)^{3.5} \quad \rho_0 = \rho \left(\frac{T_0}{T} \right)^{2.5} \quad a_0 = a \left(\frac{T_0}{T} \right)^{0.5}$$

$$Ma \text{ as a function of } A: \frac{1}{Ma} \left(\frac{(1 + 0.2Ma^2)^3}{1.728} \right) - \frac{A}{A^*} = 0 \quad A^*_{\text{max}} = \frac{\dot{m}_{\text{max}} \sqrt{RT_0}}{0.6847315 p_0} \quad \dot{m} = \rho A v$$

Property changes immediately across the shock (1 → 2):

$$Ma_2^2 = \frac{2 + 0.4Ma_1^2}{2.8Ma_1^2 - 0.4} \quad \frac{p_2}{p_1} = \frac{2.8Ma_1^2 - 0.4}{2.4} \quad \frac{\rho_2}{\rho_1} = \frac{2.4Ma_1^2}{2 + 0.4Ma_1^2}$$

$$\frac{p_{02}}{p_{01}} = \frac{\rho_{02}}{\rho_{01}} = \left(\frac{2.4Ma_1^2}{2 + 0.4Ma_1^2} \right)^{3.5} \cdot \left(\frac{2.4}{2.8Ma_1^2 - 0.4} \right)^{2.5} \quad \frac{A_2^*}{A_1^*} = \frac{Ma_2}{Ma_1} \cdot \left(\frac{2 + 0.4Ma_1^2}{2 + 0.4Ma_2^2} \right)^3$$