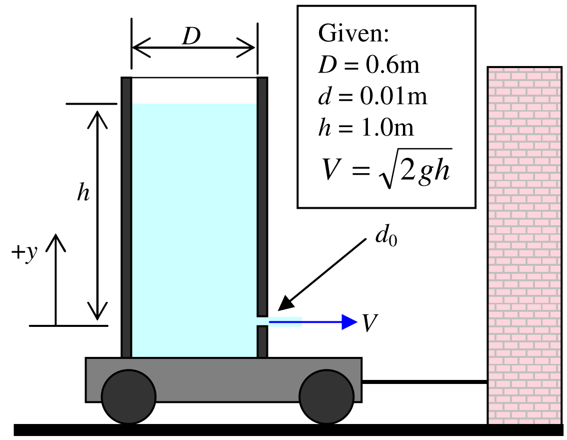
Time dependence: Recall this earlier problem from class:



Using the integral form of momentum conservation, we discovered that the tension in the rope was:

$$T = \frac{1}{2}\rho g \pi d^2 y(t)$$

Obviously, the depth y is a function of time. We could now also apply the *mass* conservation equation to this problem:

$$0 = \frac{d}{dt}\int_{CV} \rho dV + \int \rho \vec{V} \cdot d\vec{A} \, .$$

Factoring out ρ, and substituting for the known $\vec{V}$:

$$0 = \rho\left(\frac{d}{dt}\int_{CV} dV + \sqrt{2gy}\frac{\pi d^2}{4}\right)$$

Moving second term to other side…

$$\frac{d}{dt}\int_{CV}\frac{\pi D^2}{4}dy = \frac{\pi D^2}{4}\frac{d}{dt}\int_{CV} dy = \frac{\pi D^2}{4}\frac{dy}{dt} = -\sqrt{2gy}\frac{\pi d^2}{4} \rightarrow \frac{dy}{dt} = -\sqrt{2gy}\frac{d^2}{D^2}\, .$$

This is a separable first order differential equation: $y^{-\frac{1}{2}}dy = -\sqrt{2g}\frac{d^2}{D^2}dt$.

Integrating both sides: $2y^{\frac{1}{2}}\Big|_{y=h}^{y} = -\sqrt{2g}\frac{d^2}{D^2}t\Big|_0^t \rightarrow 2y^{\frac{1}{2}} - 2h^{\frac{1}{2}} = -\sqrt{2g}\frac{d^2}{D^2}t$

Solving for y: $y(t) = \left(h^{\frac{1}{2}} - \sqrt{\frac{g}{2}}\frac{d^2}{D^2}t\right)^2$.

This could be combined with the above expression for tension: $T(t) = \frac{1}{2}\rho g \pi d^2\left(h^{\frac{1}{2}} - \sqrt{\frac{g}{2}}\frac{d^2}{D^2}t\right)^2$.

The depth and tension are plotted here, versus time: