Lab 1

Uncertainty in measurement

This exercise is designed to help you understand different sources of uncertainty in measurement. You will also learn how to use the programs Tracker and Excel. This is important since we will use Tracker for several labs this semester, and we will use Excel for virtually every lab, both this semester and next semester.

1.1 Introduction

Here we briefly describe what is meant by "uncertainty" in experiments, and learn about the different types of uncertainties. Many textbooks have been written on this very important subject, the presentation here only includes the most important points.

1.1.1 Experimental uncertainty: general description

Any measurement of a physical quantity (*e.g.* mass, length, time, temperature, the acceleration of gravity, etc.) consists of two things: the observed size or magnitude of the quantity being measured, and an estimate of the uncertainty, or error, associated with the measurement. Note that "error" does not here mean "mistake" but rather "lack of ultimate precision", or "uncertainty". No physical measurement we make ever yields the "real" or "actual" result, but rather an approximation. The value of uncertainty we assign to a measurement value suggests a "range" surrounding the measured value, inside of which the "real" value most probably lies. We can only estimate the value of uncertainty associated with a certain measurement based on reasonable judgment gained from experience. To have an exact value of the uncertainty would be the same as having no uncertainly, since we could use this value to compute the "actual" value.

But note a very important point: The best we can do is to measure a physical quantity using the most sophisticated measuring device we can lay our hands on. We can never know what the "actual" or "real" value of a quantity is. Since the experimental uncertainty ultimately must be estimated, we can only speak of a "probability" that the uncertainty range assigned to a measured value does indeed surround the "real" value.

A WORD OF CAUTION: In previous science courses, you may have been taught that experimental error consists in the percentage difference between your experimentally determined number and some "accepted value". This is not what we're talking about. We are speaking of uncertainty which has only to do with the measurement itself, and is an intrinsic part of the measurement process; it has nothing to do with however far away we may be from the "accepted value". This applies whenever we use the word "error" or "uncertainty" from here on. Uncertainties are internal to an experiment.

1.1.2 Experimental uncertainty: systematic and random

There are two categories of errors—systematic errors and random (or statistical) errors. A systematic error is one which consistently shifts a measurement in one direction. For example, if a meter stick is somehow "shrunk" in length, all measurements made using this stick will be too large, no matter how carefully we read the value from the stick. Systematic errors can be very difficult to detect since one is often unaware of their presence (after all, we would not know that the stick is shorter than it should be if there is no "standard" stick with which to compare). In principle, systematic errors are avoidable, and correctable if discovered.

Random, (or statistical), errors are those errors which arise when repeated measurements of the same quantity fail to give exactly the same value (which is virtually always). They arise from physical limitations on the accuracy of a measurement and from uncontrollable fluctuations in the measuring instruments and in the objects measured. These errors are a result of such things as interpolation between scale divisions on a meter, how tightly a micrometer is placed on a rod, at which point there is a color change, line voltage fluctuations, etc. Such errors are equally likely to result in measurements that larger or smaller than the "true" value. Random errors are neither avoidable nor correctable, but they can be minimized by taking sufficient care with your experiments.

1.1.3 Precision vs. accuracy

Technically speaking the terms "precision" and "accuracy" are not synonymous. The precision of an experiment is a measure of how exactly the result is determined, without reference to what that result means. It is also a measure of how reproducible the result is. The more precise an instrument is, the smaller its random uncertainty becomes. On the other hand, the accuracy of an experiment is a measure of how close the result of the experiment comes to some "real" value. Therefore, it is a measure of the correctness of the result. Hence, we can talk about the precision of a measurement even though the accepted value may not be known, but we can't talk about accuracy (unless an accepted value is somehow known). An experimental measurement has high precision if it has small random errors and has high accuracy if it has small systematic errors.

1.1.4 Determining uncertainties

Experiments are often performed with the assumption that there are no systematic uncertainties present (otherwise we would correct them and repeat the experiment). So, determining the uncertainty on a measurement involves a statistical analysis of the random errors. When you report a measurement (x) with its uncertainty (Δx) you are indicating that you are rather confident that the real value lies within the experimental range defined from (x–Δx) to ($x + \Delta x$). Formally, the mathematics of probability suggests that, if you have a large number of independent measurements of the same quantity, then approximately 68% of the measurements should lie within this experimental range from ($x \pm \Delta x$) to ($x \pm \Delta x$). So if you report that the length of a certain widget is 10.0 ± 0.5 mm, then you are claiming that there is a 68% likelihood that the true length of the widget is between 9.5 mm and 10.5 mm. For the purpose of our introductory physics labs for the next two semesters, we will use one of the following two methods for determining random uncertainties.

Method One: Estimating the uncertainty from a single measurement or from a small number of measurements

If only one or two measurements of a quantity are made, all one can do is estimate the uncertainty in that quantity based on consideration of the nature of the system, the reliability and sophistication of the equipment being used to perform the measurement, and the precision of the definition of the quantity being measured. In estimating the value of uncertainty one should ask questions such as: "What is the smallest scale division on my ruler?", "If I 'jiggle' the apparatus, does it return to the same starting point?", "Is my friend likely to get a similar number when they measure it?", "Will I get the same result if I come back after lunch and perform the measurement again?", etc. This estimation takes practice, and there is no one correct value for the estimated uncertainty, nor is there a set of clear-cut rules one can follow. For this reason, estimating uncertainties is often loosely referred to as an art, and we will get plenty of practice estimating errors this semester.

Method Two: Uncertainty from appropriate fits of graphical data

Very often in physics lab, we will be graphing data using Excel. Often (but not always), the data will lie along a straight line, and we will use a method known as "least-squares" to determine the slope and intercept of a line which best fits the data. We will learn how to use Excel to determine the "most likely" values for the slope and intercept, and also how to use Excel to estimate the statistical uncertainty in the slope and intercept. When using Excel to determine the uncertainties in this way, we can be reasonably confident that the uncertainty represents the standard deviation. This means that we can say with approximately 68% confidence that the actual value lies within one standard deviation of the measured value.

1.1.5 Reporting results and uncertainties: "Presentation format"

Suppose you measure a cart to have a mass of 457.57 g, and you estimate the uncertainty to be 2.43 g. How should you report this? Uncertainties are estimates of our ignorance, so it is silly to claim that an uncertainty is precisely known (even when it is calculated using the standard deviation method.) Therefore we report uncertainties to no more than two significant figures. (Note that during your intermediate calculations, you should keep all of the significant figures, but when you report the final result, you should report no more than two significant figures.). The uncertainty in the mass of the cart above is, then, rounded to 2.4 g. Since the measurement can't be known to better than this uncertainty, the measurement is rounded to the same decimal place as the last digit in the uncertainty. So in this example, the mass is rounded to the "tenths" place: 457.6 g. The appropriate way to report this result is 457.6 ± 2.4 g. Note that the two numbers are reported to the same decimal place, but have a different number of sig-figs. You may never use different units (or even exponents of units) for the measurement and its uncertainty. In other words, 1.564 m $\pm$ 2.3 cm is unacceptable, but 156.4 ± 2.3 cm is acceptable.

Other examples

Pay special attention to the use of scientific notation… you must make sure to place the decimal point after the first digit in the value and then round the uncertainty to the same power of ten as

Measurement	Uncertainty	Report
0.005713 mm	0.001242 mm	$(5.7 \pm 1.2) \times 10^{-3}$ mm, or 5.7 ± 1.2 μm
35.6239 m/s	0.03423 m/s	35.624 ± 0.034 m/s
1759.21 g	782.3 g	1760 ± 780 g or 1.76 ± 0.78 kg
0.00057948 m	3.212×10^{-5} m	$(5.79 \pm 0.32) \times 10^{-4}$ m

Table 1.1: Several examples of how to report the results of a measurement properly using "presentation format".

the value. Also, don't forget to include the units after the uncertainty. Be sure that you can follow these rules every time, all the time! Some examples are shown in Table 1.1.

Summary: "Round the uncertainty to two significant figures, then round the measurement to the same power of ten as the uncertainty." More details on presentation format can be found in the introductory pages of this lab manual

1.1.6 Comparing experimental results

Often you may need to compare the results of different measurements of the same quantity to see whether they agree. Measurements are said to "agree" if their experimental ranges (from x–Δx to $x + \Delta x$) overlap.

For example, suppose Terry, Kerry, and Mary individually measure the length of a stick. Terry reports the length to be 12.02 ± 0.10 cm, Larry gets 12.21 ± 0.41 cm, and Mary gets 11.801 ± 0.052 cm. Terry's measurement (11.92 cm to 12.12 cm) overlaps with Kerry's (11.80 cm to 12.62 cm), but not with Mary's (11.749 cm to 11.853 cm). So Terry's measurement is consistent with (or agrees with) Larry's measurement, but it does not agree with Mary's measurement.

Occasionally you may want to compare your result with some "accepted" value. Ultimately, comparison between the measured value and the "accepted" value consists in seeing whether the "accepted" value lies within the region of experimental error or uncertainty surrounding your measured value. For a given measurement, you might have a large uncertainty (Δx) and so even a "bad" measurement could agree with the accepted value. Hopefully, your measurement will have small uncertainties and still agree with the "accepted" value. If your measured value, plus or minus the uncertainty, does not contain the accepted value, your result does not agree with the accepted value, even though you might feel it is quite close in an absolute sense.

Well, that's a lot to swallow… Let's see how this works in an actual experiment.

1.2 Experiment

There are two parts to this experiment which will introduce you to the two primary ways that we will be determining uncertainties. The first part will require you to *estimate* an uncertainty and in the second part you will determine the uncertainty from the *slope of a graph* using `LINEST` on Excel.

1.2.1 Part I: Estimating uncertainties

In this part of the lab, your lab instructor will ask you to make a single measurement of the period of an oscillating system. You should also estimate the uncertainty in your measurement. Now repeat this measurement 20 times and use Excel to calculate the average (use `AVERAGE()`) and standard deviation σ (use `STDEV()`) of your results. How close was your first measurement to the average? How close was your estimate of the uncertainty to the standard deviation? What percentage of your measurements fell within $\pm 1\sigma$ of the average value? What percentage of your measurements fell within $\pm 2\sigma$? What about $\pm 3\sigma$? Your lab instructor will tabulate all of the measurements made by all the students in the lab. Was your measured value and estimated uncertainty consistent with the other measurements?

1.2.2 Part II: Uncertainties from graphical data

Theory: Constant velocity

As an object moves through space, its position will change in time. If we measure its progress, we can then make a plot of its position as a function of time. Figure 1.1 shows such a plot of an object moving with *constant velocity*.

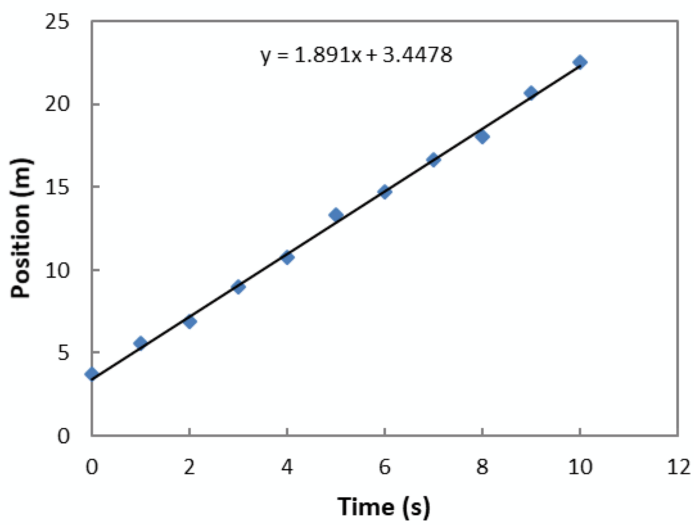

Figure 1.1: If an object moves with constant velocity, then its position versus time graph is a straight line.

We can see that these data points (the diamonds) roughly form a straight line. By adding a best-fit straight line (the solid line) to the data, we now have a mathematical object which best represents an equation that could predict the position of the object at any point in time.

You might be wondering what the two fitting parameters of the straight line represents physically. Let's find out what the slope and intercept of this line are. First, let's consider the equation of motion for an object moving at constant velocity.

$$x(t) = x_0 + vt \tag{1.1}$$

Recall that the standard algebraic form for a straight line is:

$$y = mx + b \tag{1.2}$$

Thus, considering these two equations 1.1 and 1.2, we find that if the position is chosen to be the dependent variable and time is chosen to be the independent variable, the slope will correspond to the velocity and the intercept to the initial x position.

Experiment

Your lab instructor will provide a video of a moving object. Download this video file onto your computer. We will be using the free program Tracker to analyze this video frame-by-frame. You can download Tracker at `https://www.physlets.org/tracker/` (at press time this is version 6.0.9). On Tracker, click on Video/Import and find your video file on the desktop. Use the blue arrow buttons near the bottom of the screen to find the first frame you want to track. Right click on the little black upward pointing triangle on the left side and select "Set Start Frame to Slider". Use the blue arrow buttons to find the last frame you want to track. Right click on the little black upward pointing triangle on the right side and select "Set End Frame to Slider". Now your video will only show those frames you care about. Go to Track / New / Point Mass. Hold down the shift key and click on the object on each frame. Try to be as careful as possible to click on the same point on the object at each frame.

Once you have done all the frames, click on the blue tape measure icon at the top of the screen. Select "New → Calibration Tape". Drag the tape to the edges of a fixed object in the video whose length is known, then type in the length of the object, in cm in the little box that appears. This will set the distance scale for Tracker.

Next click on the axes icon. Move the origin so that it is somewhere on the motion axis. Make sure that the x-axis lines up motion axis. If it does not, you can rotate the coordinate system by clicking out further on the x-axis and rotating the coordinate system so that the x-axis matches up with the motion axis.

Now copy the data that appears on the right hand side of the screen and paste it into Excel. Once the data is copied into Excel, make sure you redefine the numbers in the time column so that they match the frame rate of the camera. Use Excel to make a graph of position vs. time, and format it according to the instructions provided earlier in this lab manual. Use `LINEST` to fit your position vs time graph with a straight line and determine the values and uncertainties for the slope and intercept of this line. You will learn soon that the slope of a position vs. time graph represents the velocity. Determine the velocity of the particle, along with its uncertainty.

1.3 Questions

1. Does the uncertainty from `LINEST` correspond to the random uncertainty or the systematic uncertainty?
2. Are there any systematic uncertainties in this experiment that you think might be relevant?