

Series

$$f(x+h) = f(x) + hf'(x) + \frac{hf''(x)}{2!} + \frac{hf'''(x)}{3!} + \dots$$

$$\sum_{i=0}^n ar^i = \frac{a(1-r^{n+1})}{1-r} \quad \sum_{i=0}^{\infty} ar^i = \frac{a}{1-r}$$

$$\sum_{i=1}^n ar^i = \sum_{i=0}^n ar^i - a \quad \sum_{i=1}^{\infty} ar^i = \frac{ar}{1-r}$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \quad \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n = 1 + px + \frac{p(p-1)}{2!} x^2 + \frac{p(p-1)(p-2)}{3!} x^3 + \dots$$

Coordinate Systems

$$x = r \cos(\theta)$$

$$y = r \sin(\theta)$$

$$dA = r dr d\theta$$

$$ds = \sqrt{1+r^2 \left(\frac{d\theta}{dr} \right)^2} dr$$

$$dV = r dr d\theta dz$$

$$dA = r d\theta dz$$

$$ds^2 = dr^2 + r^2 d\theta^2 + dz^2$$

$$x = r \sin(\theta) \cos(\phi)$$

$$y = r \sin(\theta) \sin(\phi)$$

$$z = r \cos(\theta)$$

$$dV = r^2 \sin(\theta) dr d\theta d\phi$$

$$dA = r^2 \sin(\theta) d\theta d\phi$$

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2(\theta) d\phi^2$$

ODE's

$$y' + Py = Q \rightarrow$$

$$I = \int P dx$$

$$y = e^{-I} \int Q e^I dx + c e^{-I}$$

Given r_1, r_2 :

$$y = A e^{r_1 x} + B e^{r_2 x} \quad (r_1 \neq r_2)$$

$$y = (Ax + B) e^{r_1 x} \quad (r_1 = r_2)$$

$$y = A e^{r_1 x} + B e^{r_2 x} \quad (r_1, r_2 = \alpha \pm i\beta)$$

$$\text{or } y = e^{\alpha x} (A \sin(\beta x) + B \cos(\beta x))$$

$$\text{or } y = C e^{\alpha x} \sin(\beta x + \gamma)$$

$$y_p = \begin{cases} C e^{cx} & \text{if } c \neq a, c \neq b \\ C x e^{cx} & \text{if } c = (a \text{ or } b), a \neq b \\ C x^2 e^{cx} & \text{if } c = a = b \end{cases}$$

Probability

$$p(AB) = p(A)p_A(B)$$

$$p(A+B) = p(A) + p(B) - p(AB)$$

$$P(n, r) = \frac{n!}{(n-r)!}$$

$$C(n, r) = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

$$\sigma^2 = \sum_i (x_i - \mu)^2 f(x_i) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$$

$$\text{Binomial: } f(x) = C(n, x) p^x q^{n-x}$$

$$\text{Normal: } f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\sigma^2 \approx \frac{1}{n-1} \sum_{i=1}^n (x_i - \mu)^2$$

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

$$\text{Poisson: } P_n = \frac{\mu^n}{n!} e^{-\mu}$$

Fourier

$$f(x) = \frac{1}{2} a_0 + a_1 \cos\left(\frac{1\pi x}{l}\right) + a_2 \cos\left(\frac{2\pi x}{l}\right) + \dots + b_1 \sin\left(\frac{1\pi x}{l}\right) + b_2 \sin\left(\frac{2\pi x}{l}\right) + \dots$$

$$a_n = \frac{1}{l} \int_{-l}^l f(x) \cos\left(\frac{n\pi x}{l}\right) dx$$

$$b_n = \frac{1}{l} \int_{-l}^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

$$f(x) = \int_{-\infty}^{+\infty} g(\alpha) e^{i\alpha x} d\alpha$$

$$g(\alpha) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(x) e^{-i\alpha x} dx$$

Derivatives & Integrals

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \dots$$

$$ds = \sqrt{dx^2 + dy^2}$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx$$

$$\bar{x} = \frac{\int x dm}{\int dm}$$

$$I = \int r^2 dm$$

Complex Numbers

$$z = x + iy = r e^{i\theta}$$

$$z = r \cos(\theta) + ri \sin(\theta)$$

$$\sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sinh(z) = \frac{e^z - e^{-z}}{2}$$

$$\cosh(z) = \frac{e^z + e^{-z}}{2}$$

$$Z_R = R$$

$$Z_L = \omega L i$$

$$Z_C = -(\omega C)^{-1} i$$

Inertia Tensors

$$I_{xx} = \sum_i m_i (y_i^2 + z_i^2) = \int (y^2 + z^2) dm$$

$$I_{xy} = -\sum_i m_i x_i y_i = -\int xy dm$$

Linear Algebra

$$\text{Active Rotation Matrix: } \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Passive Rotation Matrix: } \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Vectors

$$\bar{\nabla} = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$$

$$\nabla^2 \phi = \bar{\nabla} \cdot \bar{\nabla} \phi$$

$$\bar{V} = \bar{\nabla} \phi$$

$$\psi = \int v_x dy = -\int v_y dx$$